Certification of High-Dimensional Entanglement in Ultracold Atom Systems

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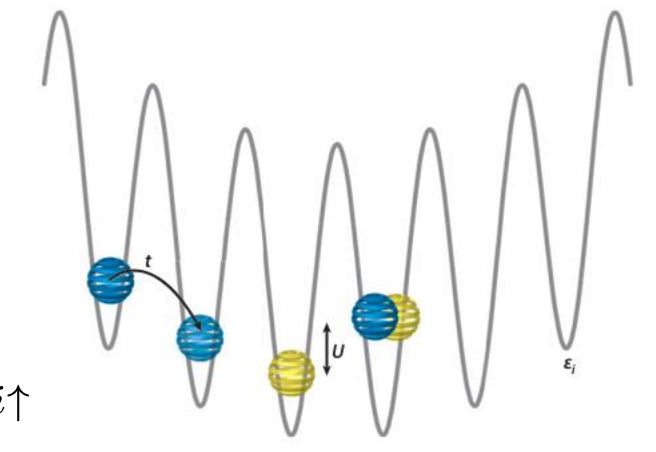
Motivation

- Entanglement as physical resource of quantum communication & computing protocols → Need for certifying entanglement in quantum devices
- Entanglement dimension and entanglement spectrum are physically relevant properties e.g. in condensed matter physics and quantum statistical mechanics
- Cold Atoms are highly developed quantum simulator, but entanglement certification and quantification is challenging

Targeted Systems

- Applicable to cold atoms in optical lattices
- Requires position and momentum basis readout
- Single particle resolved imaging



$$H = -J\sum_{\sigma}\sum_{i,j}(\hat{c}^{\dagger}_{i,\sigma}c_{j,\sigma} + \text{h.c.}) + U\sum_i \hat{n}_{i\downarrow}\hat{n}_{i\uparrow}$$

[T. Esslinger. *Ann. Rev. of Cond. Matt. Phys.*, 2010]

Entanglement & Fidelity Bounds for Bipartite Systems

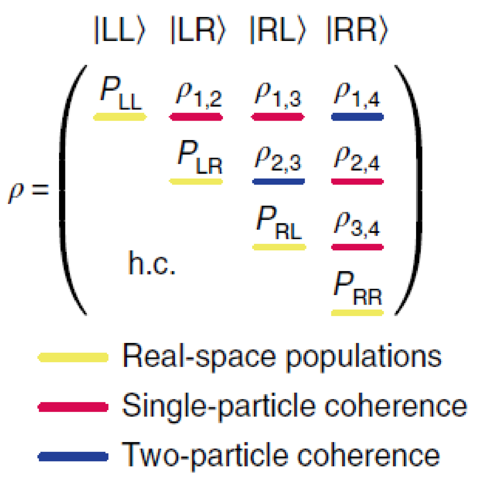
- All bipartite Systems have unique minimal basis choice (**Schmidt Decomposition**):

$$|\psi\rangle_{AB} = \sum_{i=1}^{k}\lambda_i \cdot |i\rangle_A \otimes |i\rangle_B \xrightarrow[\text{state}]{\text{max. entangled}} |\Psi\rangle_{MES} = \frac{1}{\sqrt{N}}\cdot\sum_{m=1}^{N}|mm\rangle$$

- Use fidelity to state Ψ to bound **entanglement dimension k**[1]:

$$F(\rho,\Psi) \leq B_k(\Psi) = \sum_{i=1}^{k}\lambda_i^2, \qquad \lambda_i > \lambda_{i+1}$$

Entanglement dimension of the experimental state ρ



[A. Bergschneider. *Nat.Phys.*, 2019]

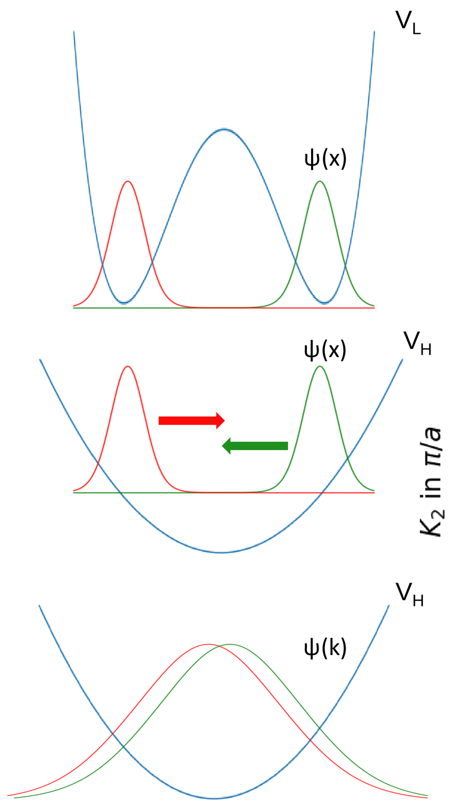
$$F(\rho,\Psi_{MES}) = \frac{1}{N}\sum_{m=1}^{N}\langle mm|\,\rho\,|mm\rangle + \frac{1}{N}\sum_{\substack{m,n=1\\ m\neq n}}^{N}\langle mm|\,\rho\,|nn\rangle$$

Real-space populations
- Readily accessible
- In situ measurement of atom positions
- Fluorescence imaging

Two-particle coherences
- Not directly accessible through in situ measurement
- Measure in 2nd basis → Exploit momentum correlations[2]

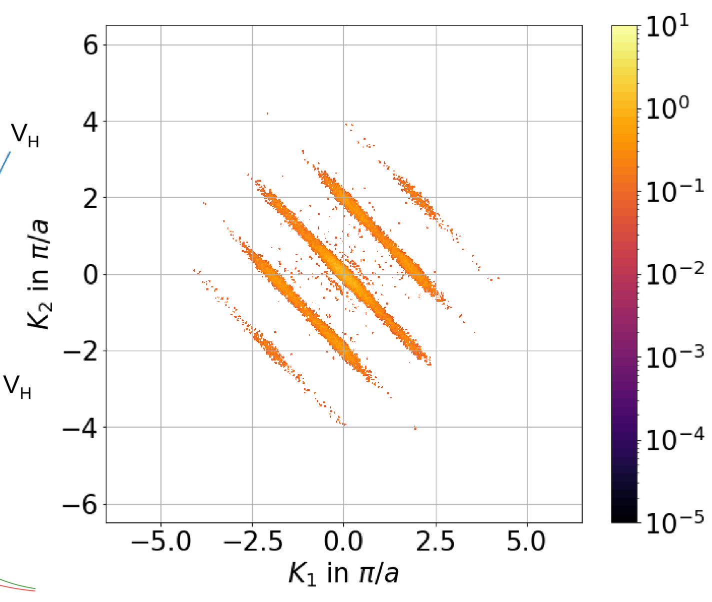
Momentum Distribution Frequency Decomposition

Particles localized on lattice sites → Propagate for T/4 in weak harmonic potential V_H → Effective Fourier transformation[2]



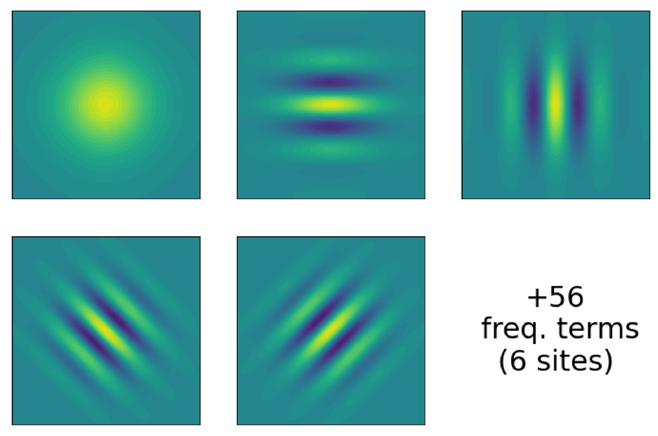
$$\langle k_1 k_2\rangle = |\tilde{w}(k_1,k_2)|^2\sum_{\alpha_1,\alpha_2\in\mathbb{Z}}\text{Re}(c_{\alpha_1,\alpha_2})\cdot\cos(\alpha_1 k_1 + \alpha_2 k_2) + \text{Im}(c_{\alpha_1,\alpha_2})\cdot\sin(\alpha_1 k_1 + \alpha_2 k_2)$$

$$c_{\alpha_1,\alpha_2} = \sum_{(i,j)\in C}\frac{\rho_{ij}}{2}, \qquad C = \{(i,j)\in\mathbb{N}^2|\,|i\rangle \equiv |i_1,\, i_2\rangle,\, |j_1,\, j_2\rangle = |i_1+\alpha_1,\, i_2+\alpha_2\rangle\}$$

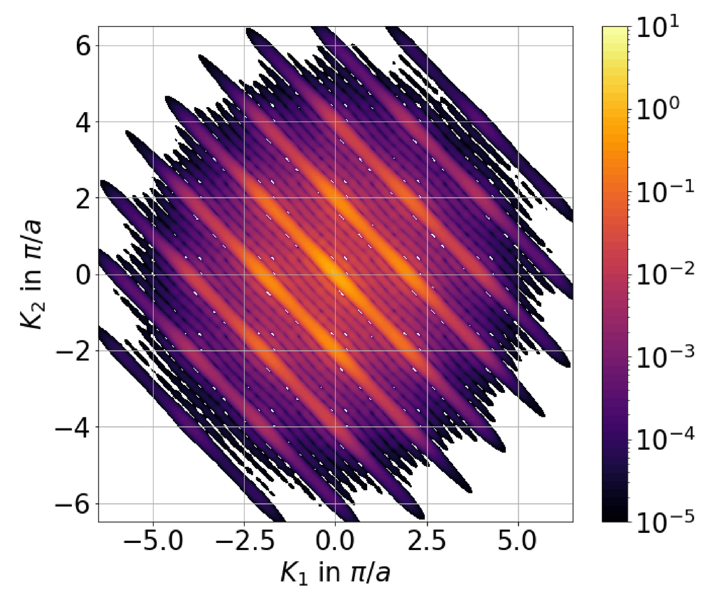


Basis coefficients are sums over coherences of experimental state ρ[1]

Expand in frequency basis



Reconstruct momentum correlations



Coherence Extraction Scheme

- Frequency basis coefficients contain coherence sums → Only same site coherences $\langle mm|\,\rho\,|nn\rangle$ relevant
- Bound unwanted coherences: $|\langle m'n'|\,\rho\,|mn\rangle| \overset{\text{CSI}}{\leq} \sqrt{\langle m'n'|m'n'\rangle\cdot\langle mn|mn\rangle}$

Accessible state populations

- Subtract differing-site two-particle coherences:

$$\frac{1}{N}\sum_{\substack{m,n=1\\ m\neq n}}^{N}\langle mm|\,\rho\,|nn\rangle \geq \frac{2}{N}\left(\sum_{\substack{i,j=1\\ i\neq j}}^{N-1}c_{iijj} - \sum_{\substack{\langle m'n'|mn\rangle\\ \text{contr. to } c_{iijj}}}\sqrt{\langle m'n'|m'n'\rangle\cdot\langle mn|mn\rangle}\right) = F_{\text{bound}}$$

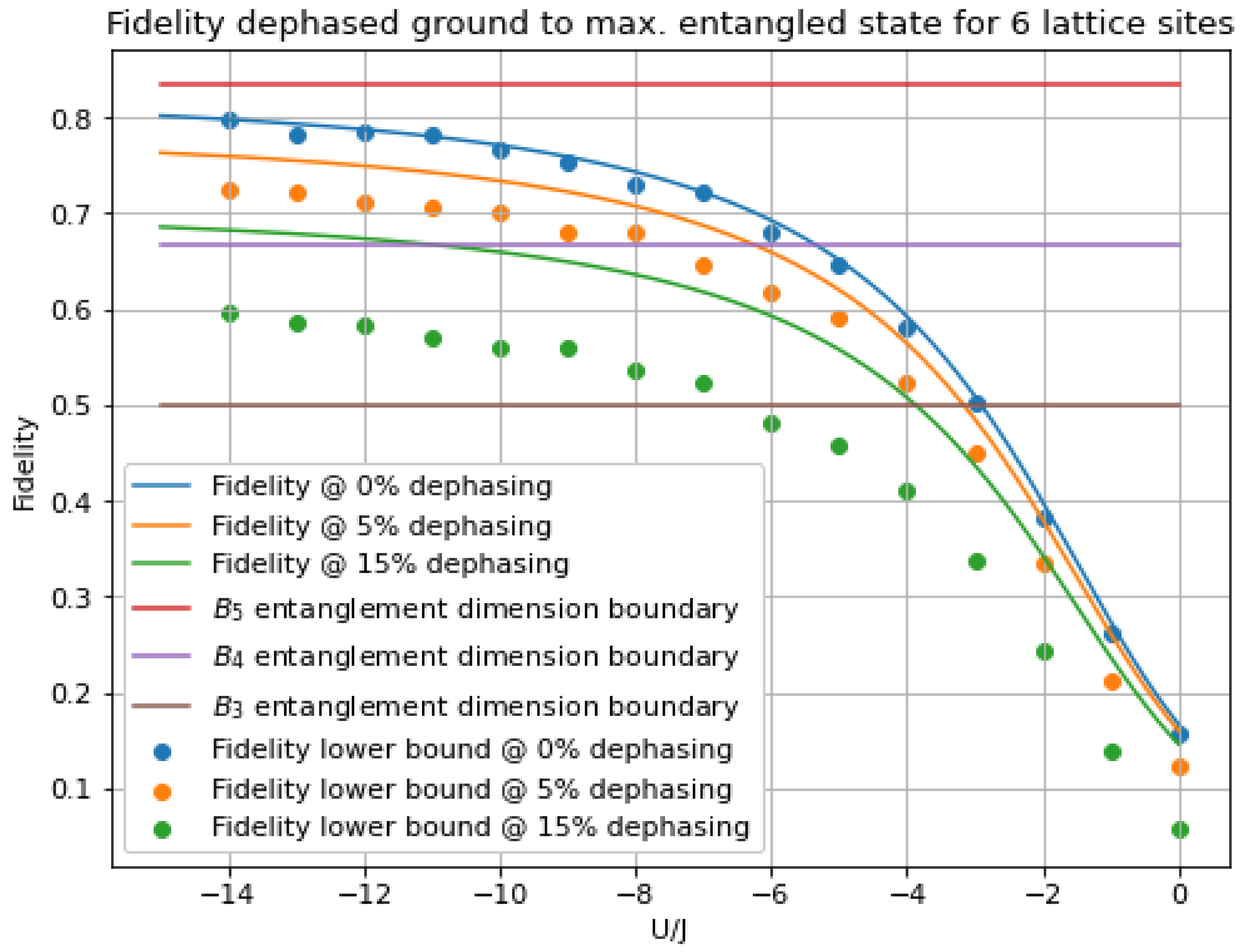
Frequency basis coefficients containing relevant coherences

Expandable single-particle and two-particle coherence bounds

- Complete fidelity bound: $\tilde{F}(\rho,\Psi_{MES}) = \frac{1}{N}\sum_{m=1}^{N}\langle mm|\,\rho\,|mm\rangle + F_{\text{bound}}$

  Such that: $\tilde{F}(\rho,\Psi_{MES}) \leq F(\rho,\Psi_{MES}) \leq B_k(\Psi_{MES})$

Simulation Results



- Up to 5-dimensional entanglement certifiable in attractive regime (6 sites)
- Bound tightness decreases for increasing dephasing / statistical noise

Next up: Tripartite Systems

- No Schmidt decomp. for general tripartite states → GHZ-like state already minimal[3]
- Algorithm extended to multipartite attractive scenarios $|\psi\rangle_{GHZ} = \frac{1}{N}\sum_{i=1}^{N}|iii\rangle$
- Challenge: Lattice depth stability → Strong localizing effect

Summary & Outlook

- Entanglement dimension bound from below by fidelity measurements
- Utilize position and momentum correlations to obtain two-particle coherences
- Simulation: Certify up to 5-dim. entanglement for dephased ground state
- Long term: True many-body regime: $D_{\text{Ent}} = N \xrightarrow[\text{per party}]{\text{d atoms}} \binom{N}{d}$

References

[1] J. Bavaresco et al. Measurements in two bases are sufficient for certifying high-dimensional entanglement. *Nature Physics*, 2018.
[2] A. Bergschneider et al. Experimental characterization of two-particle entanglement through position and momentum correlations. *Nature Physics*, 2019.
[3] A. Thapliyal. Multipartite pure-state entanglement. *Phys. Rev. A*, 1999.

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