Spreading of Correlations and Entanglement in the Long-Range Transverse Ising Chain

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1. Motivation

The dramatic effect of long-range interactions on the dynamics of quantum matter has attracted significant experimental and theoretical attention in recent years. In contrast with short-range interacting lattice models, where the spreading of correlations is limited by the well-known Lieb-Robinson bounds, sufficiently long-range interactions lead to the instantaneous propagation of information and the breakdown of the notion of causality, consistent with the absence of known Lieb-Robinson bounds. In the intermediate (quasi-local) regime between these two limits, the existence of some form of causality is strongly debated and remains a crucial outstanding problem.

2. Long-range interactions

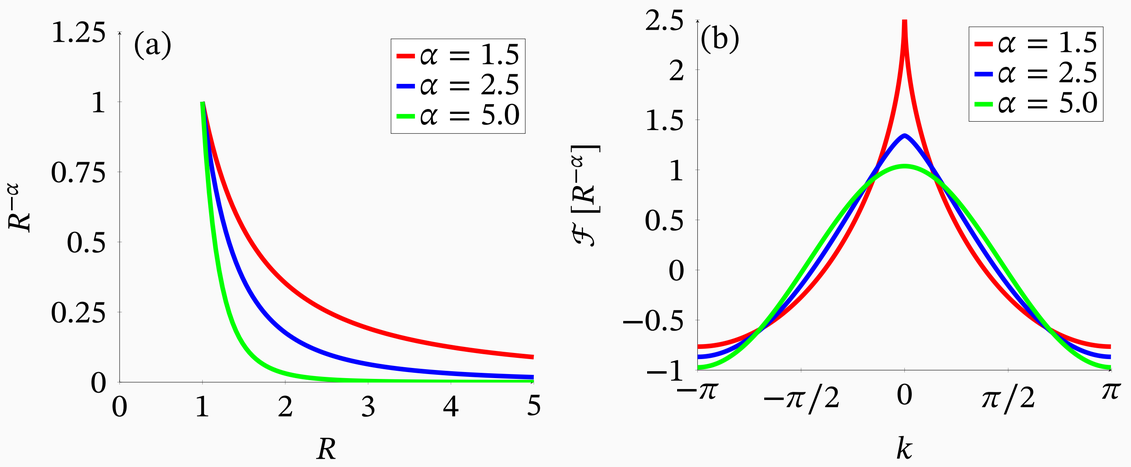


Fig. 1: (a) long-range interaction potentials. Note the regularization of the UV divergence but not the (possible) IR divergence on an infinite lattice. (b) Fourier transform of long-range interaction potential. Note the non-analyticity at $k = 0$ for $\alpha < 2$.

3. Model and approach

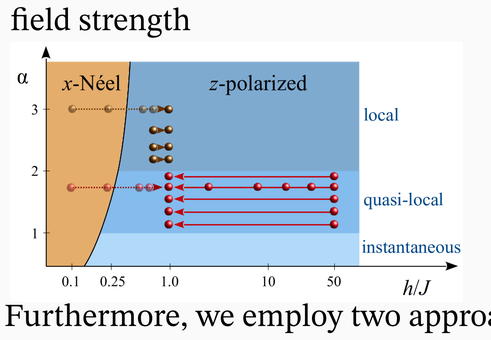
We address the emergence of causality in the quasi-local regime by studying the out-of-equilibrium dynamics of the one-dimensional transverse Ising model with algebraic long-range exchange coupling,

$$H = \sum_{R \neq R'} \frac{J}{|R - R'|^{\alpha}} S_R^x S_{R'}^x - 2h \sum_R S_R^z \,. \tag{1}$$

We induce out-of-equilibrium dynamics via two types of quenches.

1st: **Global quench**, change transverse field strength

2nd: **Local quench**, spin flip at center of ground state ($h/J = 50$)



$$|\uparrow \dots \uparrow\rangle \xrightarrow{\text{spin flip}} |\uparrow \dots \uparrow\downarrow\uparrow \dots \uparrow\rangle$$

Furthermore, we employ two approaches,

1. state of the art **tensor-network approach** via density matrix renormalization group (DMRG) and time-dependent variational principle (TDVP),
2. **analytic approach** via linear spin wave theory (LSWT) valid for $h/J \gg 1$,

and consider various experimentally accessible quantities, like spin–spin correlation functions, local magnetization and Rényi entropies.

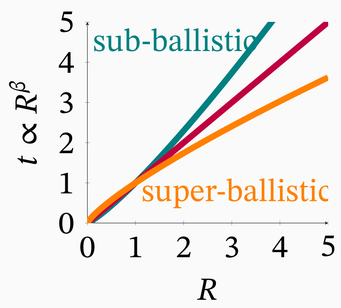


Fig. 2 Power law spreading in a t–R–diagram: sub-ballistic (teal), ballistic (red), and super-ballistic (orange). This power law is the ansatz for describing several different signal edges in long-range interacting spin models.

4. Results

We are inspecting the two-point function,

$$G_{x,z}(R,t) = G^0_{x,z}(R,t) - G^0_{x,z}(R,t=0)\,, \tag{2}$$

$$G^0_{x,z}(R,t) = \langle S_R^{x,z}(t) S_0^{x,z}(t)\rangle - \langle S_R^{x,z}(t)\rangle\langle S_0^{x,z}(t)\rangle\,, \tag{3}$$

the local magnetization $(1/2 - \langle S_R^z(t)\rangle)$, and the bipartite Rényi entanglement entropies $\mathcal{S}_n = \frac{1}{1-n}\log\{\mathrm{tr}[\hat{\rho}^n]\}$, and fit power laws ($t \propto R^{\beta}$) to their signal edges and local maxima, respectively. We corroborate our numerical results with an analytical method (LSWT) which explains them in terms of a quasi-particle picture.

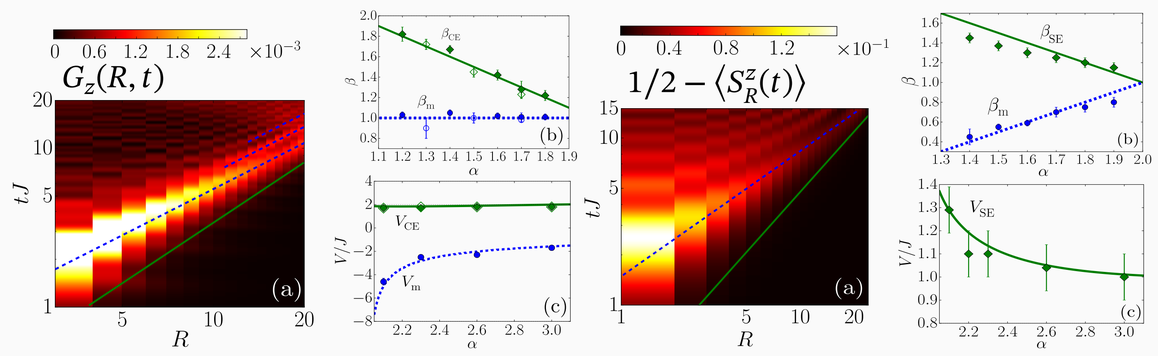


Fig. 3 (a) $G_z(R,t)$ after a global quench via TDVP simulation. Solid green line is power law fit to the correlation edge (CE), dashed blue line is fit to local maxima with dynamical exponents β_{CE} and β_m, respectively. (b) In green $\beta_{CE}(\alpha)$ from TDVP (points) and LSWT (solid line, $\beta_{CE} = 3 - \alpha$). In blue $\beta_m(\alpha)$ from TDVP (points) and from LSWT (dashed line, $\beta_m = 1$). (c) As above for spreading velocities over α in the local regime.

Fig. 4 (a) $1/2 - S_R^z(t)$ after a local quench via TDVP simulation. Solid green line is fit to the spin edge (SE), dashed blue line is fit to local maxima with dynamical exponents β_{SE} and β_m, respectively. (b) In green $\beta_{SE}(\alpha)$ from TDVP (points) and LSWT (solid line, $\beta_{SE} = 3 - \alpha$). In blue $\beta_m(\alpha)$ from TDVP (points) and from LSWT (dashed line, $\beta_m = \alpha - 1$). (c) As above for spreading velocity over α in the local regime.

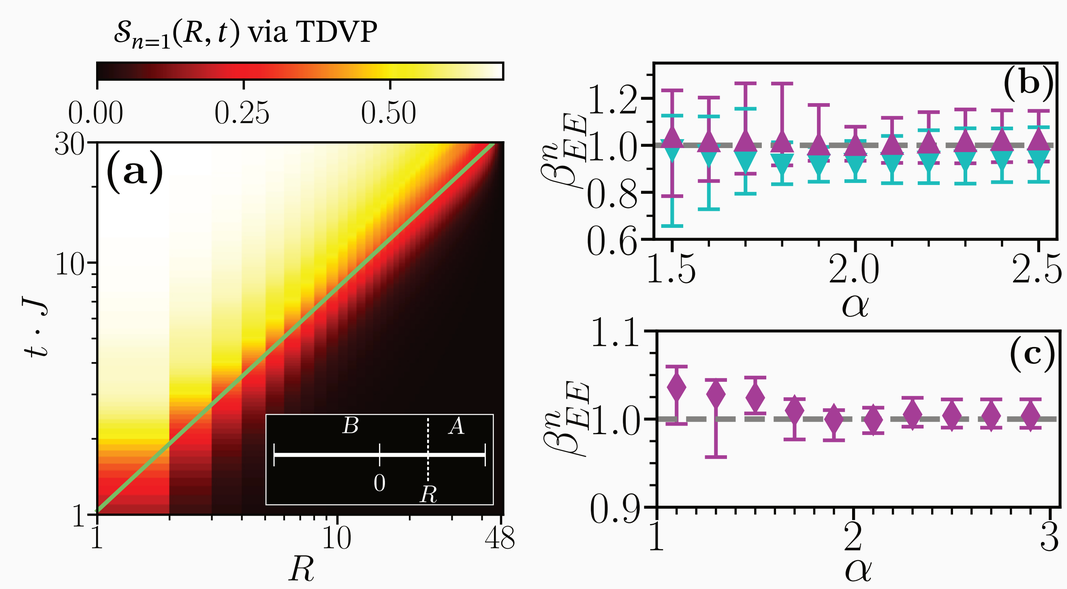


Fig. 4 (a) von Neumann entanglement entropy ($n \to 1$) via TDVP after a local quench. Solid green line: power law fit to entanglement edge (EE). (b) Fitted $\beta^{n=1}_{EE}$ over α obtained from TDVP (cyan) and LSWT (magenta) in system size $N = 96$. Grey dashed lined is LSWT prediction $\beta_{EE} = 1$ (c) Fitted $\beta^{n=1}_{EE}$ over α obtained LSWT (magenta) in system size $N = 512$. Grey dashed lined is LSWT prediction $\beta_{EE} = 1$.

Surprisingly, the EE always scales ballistically. This can be predicted by means of LSWT. We find $\rho_A = \mathrm{tr}_B(\rho) = \lambda_1(t)\,|0_A\rangle\langle 0_A| + \lambda_2(t)\,|\chi_A\rangle\langle\chi_A|$, whereas χ_A is a complex superposition of quasi-particles in the first excited manifold. Consequently, the entanglement entropy is bounded from above by $\log(2) = 0.69$. In the asymptotic limit and for not too small values of R/t, we find $\lambda_2(R,t) \propto t^{\frac{1}{2-\alpha}}\zeta\left(\frac{3-\alpha}{2-\alpha}, R\right) \sim (t/R)^{\frac{1}{2-\alpha}}$, with ζ the Hurwitz zeta function. Hence, the n-order Rényi entropy is a function of the ratio R/t, which confirms ballistic scaling for all Rényi orders.

5. Conclusion

Our results demonstrate the emergence of a weak form of causality in the quasi-local regime of long-range spin models, characterized by fundamentally non-universal scaling laws which allows us to reconcile contrasting observations in the existing literature [1, 2]. We further demonstrate that the scaling of quantum entanglement takes on a universal form with a well-defined entanglement edge which propagates ballistically for all interaction ranges we consider.

References

1. Foss-Feig, M., Gong, Z.-X., Clark, C. W. & Gorshkov, A. V. Nearly Linear Light Cones in Long-Range Interacting Quantum Systems. *Phys. Rev. Lett.* **114,** 157201 (15 2015).
2. Cevolani, L., Carleo, G. & Sanchez-Palencia, L. Protected quasi-locality in quantum systems with long-range interactions. *Phys. Rev. A* **92,** 041603(R) (2015).