Stability of optomechanical self-structured phases and dissipative solitons of cold atoms with optical feedback

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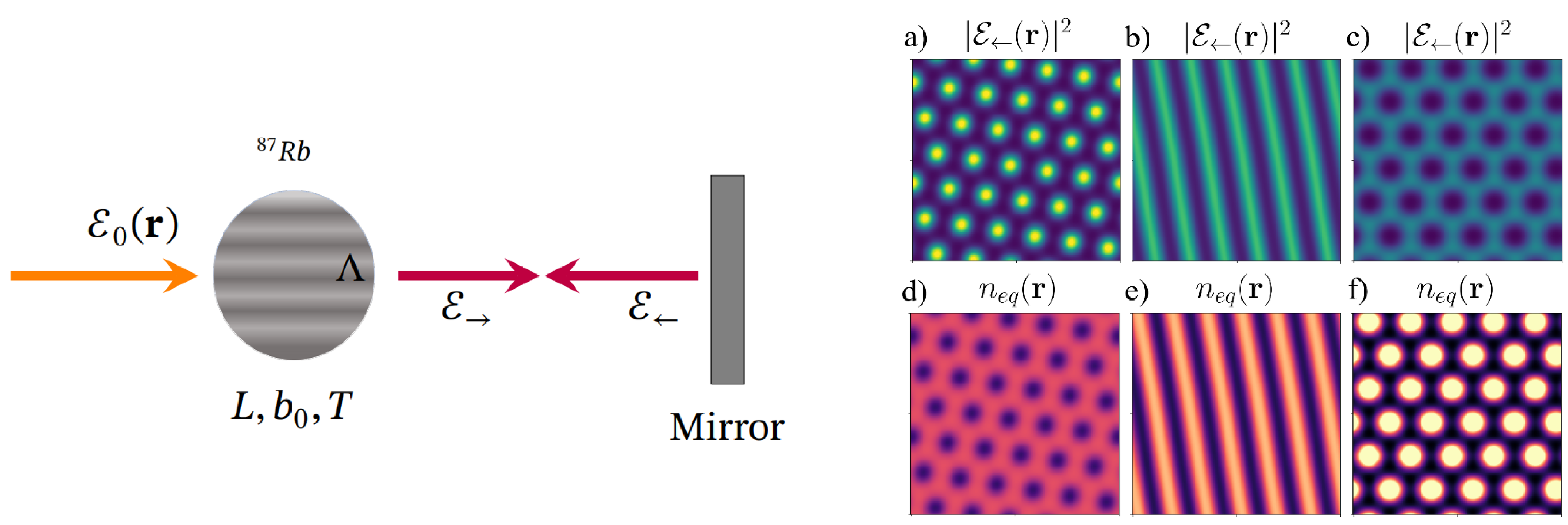
Introduction

The spontaneous emergence of density ordered states via **optomechanical forces** is a prominent feature of cold atomic systems driven far from equilibrium [1]. In transverse optical setups with **optical feedback**, the collective atomic bunching gives rise to a self-focusing Kerr-like nonlinearity, displaying modulation instabilities that result in self-organized hexagonal stuctures [2]. In this work, we show instead that the optomechanical self-structuring displays a richer **structural transition behaviour**, characterized in terms of three crystalline phases, i.e., **hexagonal, roll/stripe and honeycomb** ($\mathbf{H}^+, \mathbf{R}, \mathbf{H}^-$) [3]. Moreover, the subcriticality of the $\mathbf{H}^+$ phase allows the existence of a feedback soliton functioning as a self-sustained dark atomic trap.

The model (Single-Feedback-Mirror)

We consider a thermal cloud of two-level atoms at constant temperature T, where the atomic motion is overdamped by means of optical molasses beams. In this regime, the medium dynamics is described by the **Smoluchowski equation** (dipole force + spatial diffusion) for the atomic density distribution $n(\mathbf{r}, t)$. We denote with χ the linear susceptibility of the cloud.



$$\chi = \frac{b_0 \Delta}{2(1+\Delta^2)}, \qquad \partial_t n(\mathbf{r},t) = \sigma D_\mathbf{r} \nabla_\mathbf{r} \cdot [n(\mathbf{r},t)\, \nabla_\mathbf{r}\, s(\mathbf{r},t)] + D_\mathbf{r} \nabla^2_\mathbf{r}\, n(\mathbf{r},t) \qquad \sigma = \frac{\hbar \Gamma \Delta}{4 k_B T}$$

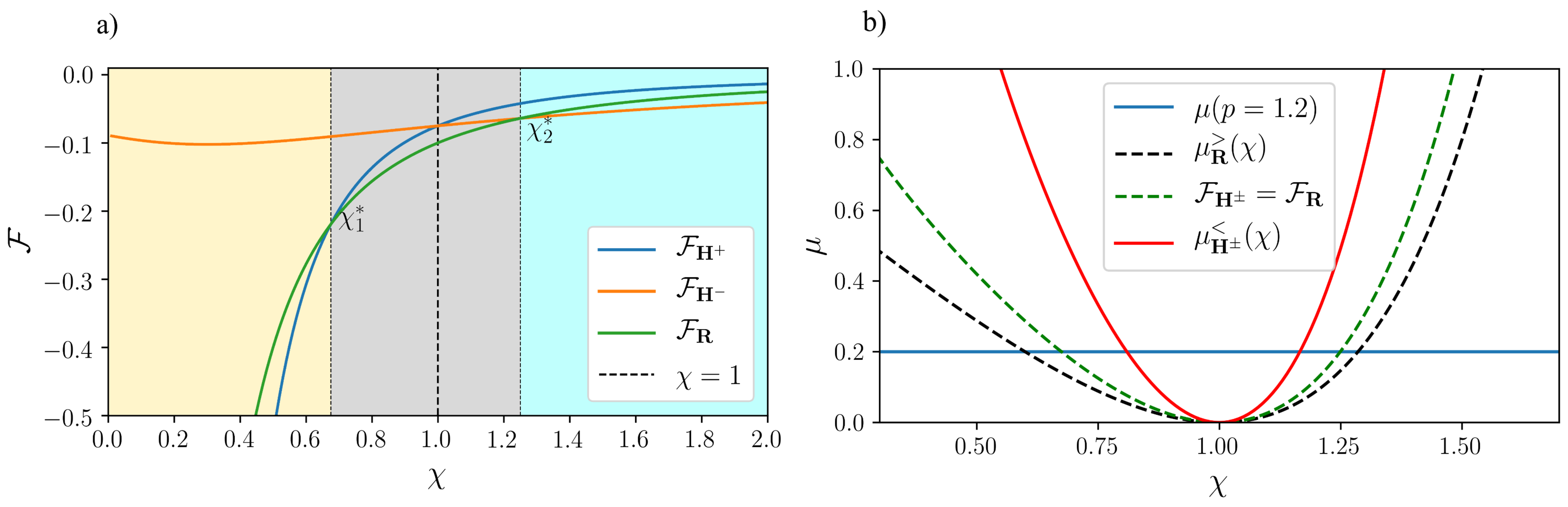
where b_0 is the optical density at resonance, Δ is the light-atom detuning, Γ the decay rate and $s(\mathbf{r}, t)$ the saturation intensity. The self-structured phases above are numerically obtained by solving the feedback loop at fixed $b_0 = 110$ and $T = 300\,\mu$K. (a), (d) $\mathbf{H}^-$ phase at $\Delta = 25$. (b),(e) $\mathbf{R}$ phase at $\Delta = 55$. (c), (f) $\mathbf{H}^+$ phase at $\Delta = 90$.

Hexagon-Roll competition

The atom-field system can be formally reduced to a single closed equation for the density perturbation $\delta n(\mathbf{r}, t)$. We expand up to third order in δn and derive the solvability conditions, leading to the amplitude equations [4]:

$$\partial_t A_i = -\frac{\delta \mathcal{F}[\{A_i\}]}{\delta A_i^*}, \qquad \mathcal{F}[\{A_i\}] = -\mu \sum_{i=1}^{3} |A_i|^2 - \lambda \left(A_1^* A_2^* A_3^* + \text{c.c.}\right) + \frac{\gamma_2}{2} \sum_{i,j=1}^{3} |A_i|^2 |A_j|^2 + \frac{\gamma_1}{2} \sum_{i=1}^{3} |A_i|^4$$

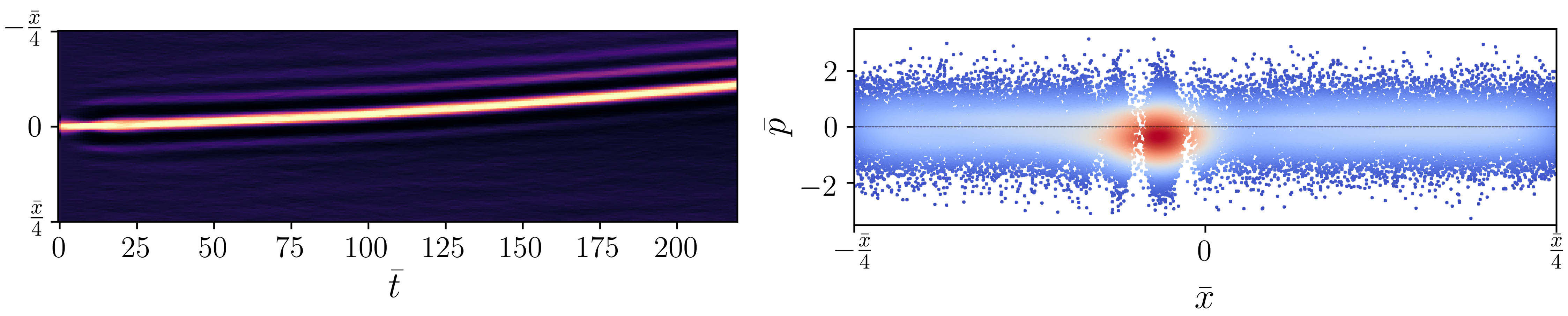
Where $\mathcal{F}[\{A_i\}]$ is the **free energy functional**. Its dependence on χ for the different phases and the corresponding minima are shown in (a). The intersections in (b) determine the observed **phase boundaries** in good agreement with numerical simulations.



Note that for $\chi = 1$, we have $\lambda = 0$, and the system recovers the **inversion symmetry**.

Optomechanical transport induced by OAM

We perform a set of **1D particle dynamics simulations** describing the formation and angular (rotational) dynamics of the dark blue-detuned feedback soliton in the presence of an OAM carrying pump [5]:



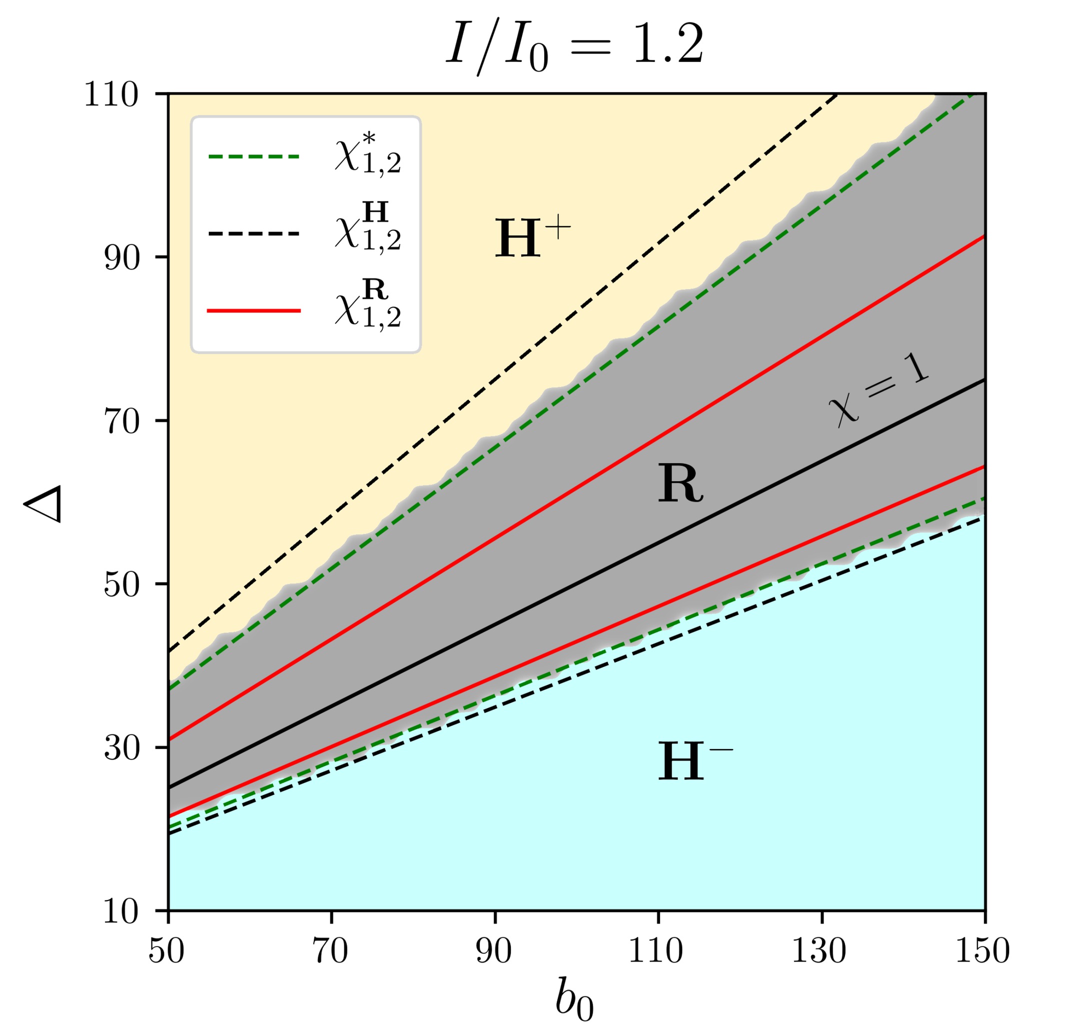
Applying a linear phase on the input pump generates angular drift [3]. The initially prepared density peak reaches steady state motion and non-zero average angular momentum (**mass current**) of the atoms trapped in the soliton region (highlighted in red).

Phase diagram

To characterize the transitions between phases we span the space (Δ, b_0) within the experimentally achievable ranges $\Delta \in [10, 110]$ and $b_0 \in [50, 150]$ [2]. We start with a perturbed $\mathbf{R}$ state and iterate the loop long enough to reach stabilization with the equilibrium density:

$$n_{\text{eq}}(\mathbf{r}, t) = \frac{\exp[-\sigma s(\mathbf{r}, t)]}{\int_\Omega d^2\mathbf{r}_\perp \exp[-\sigma s(\mathbf{r}, t)]},$$

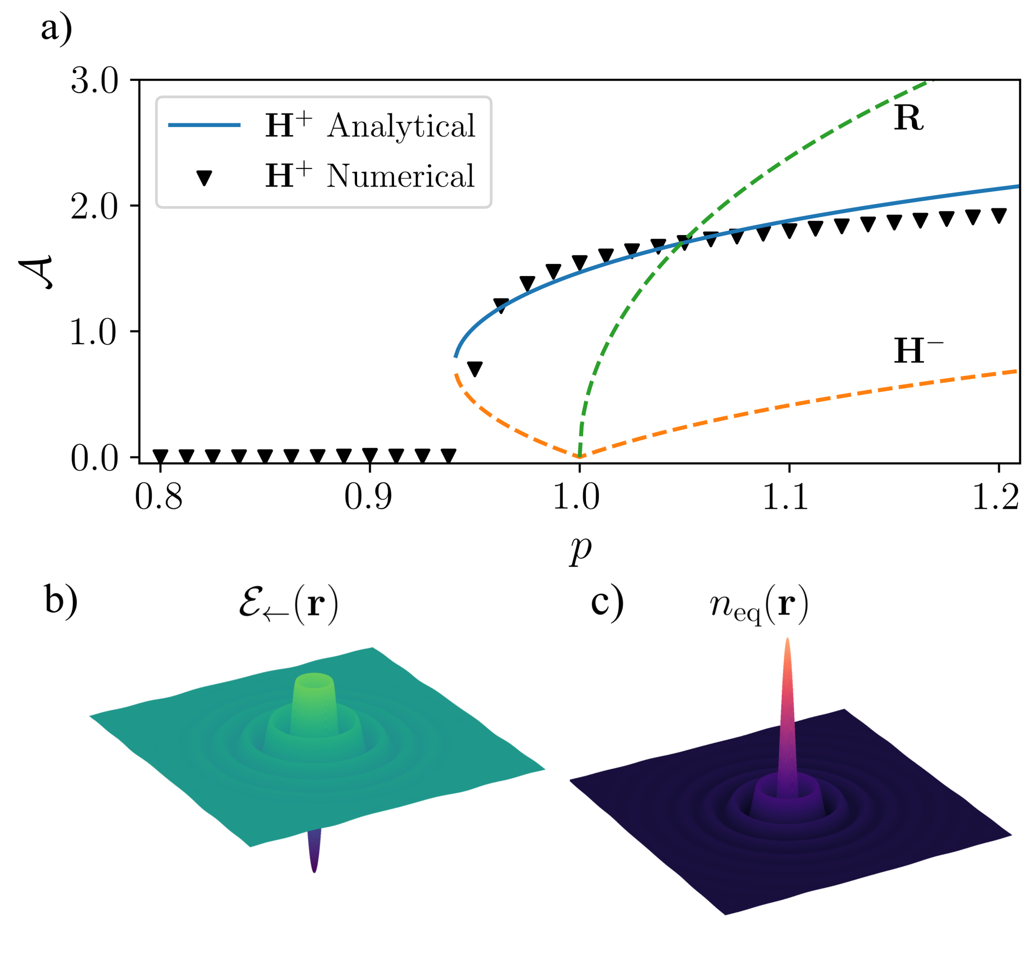
We obtain the following **phase diagram** at fixed distance from the instability threshold I_0:



We report a stability domain of $\mathbf{R}$ states sandwiched between two disjoint $\mathbf{H}^\pm$ regions, separated by lines of constant χ. Transitions between phase boundaries are addressed in terms of a weakly nonlinear espansion based on the **real Ginzburg-Landau amplitude equations**.

Dark self-sustained traps

The optomechanical nonlinearity displays a **structural phase transition** between hexagonal states $\mathbf{H}^- \rightarrow \mathbf{H}^+$ of $n(\mathbf{r}, t)$. This allows the existence of dark, blue-detuned solitons. (See below for $b_0 = 50$, $\Delta = 80$, $\sigma \approx 78.3$).



where $p = I/I_0$. Such a soliton configuration at large detuning ($\Delta \gg 1$) enables the realization of a controllable **dark, self-sustained dipole trap** for cold and ultracold atoms.

Future directions

- Structural phase transitions in the case of a **quantum degenerate gas**?
- Role of **dissipation** in our system.
- Connections with the concept of **quantum droplet**.

References

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