Strong correlations in lossy one-dimensional quantum gases:

from the quantum Zeno effect to the generalized Gibbs ensemble

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Laboratoire de Physique Théorique et Modèles Statistiques

Laboratoire d'Excellence Physique : Atomes Lumière Matière

Strong Dissipation Inhibits Losses and Induces Correlations in Cold Molecular Gases

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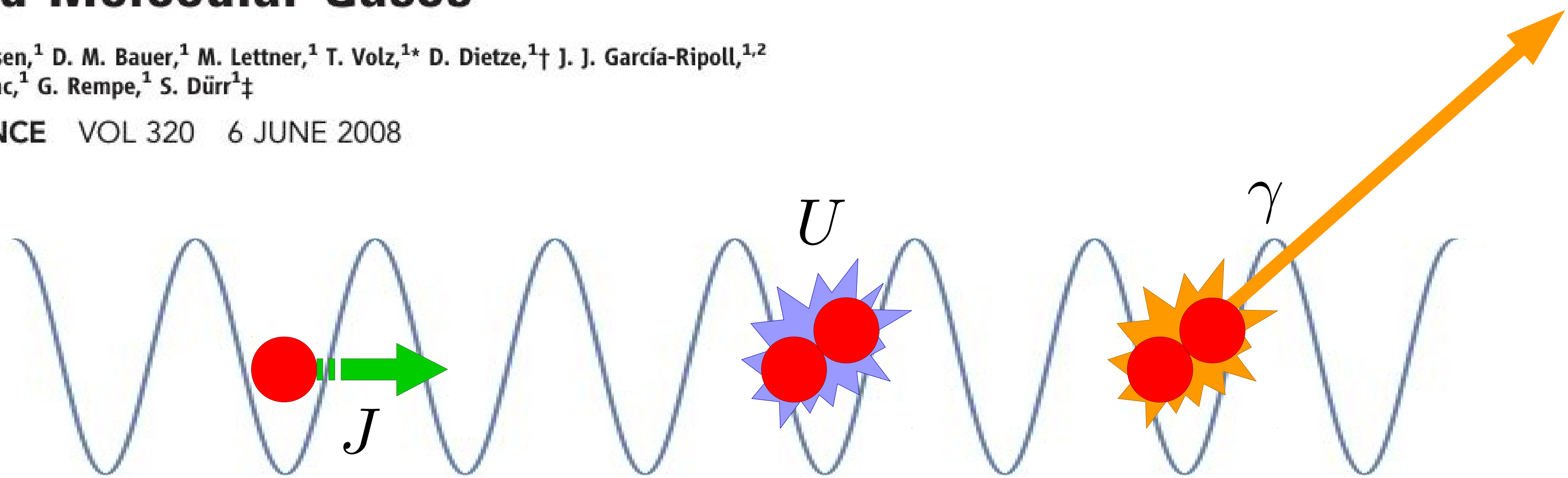
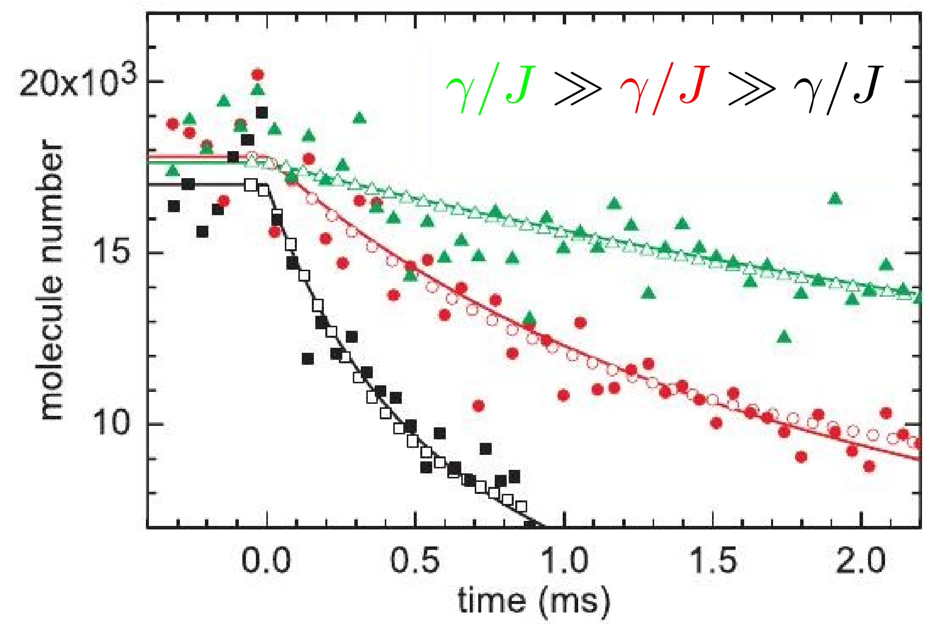


Dissipative (lossy) Bose-Hubbard model

- Ratio *U/γ* fixed
- Ratio *J/γ* tunable
- Experimental study: strong dissipation

Intuitive ideas

- Existence of a long-lived space of bosons with at most one boson per site
- Lifetime increases with *γ*
  - *Quantum Zeno effect*



Take-home message:
strong dissipation creates fermionized bosons

The master equation

Open quantum system because of atomic losses

- description in terms of a density matrix through a **Lindblad master equation**

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \mathcal{L}[\rho(t)] = -\frac{i}{\hbar}[H,\rho(t)] + \sum_j L_j \rho(t) L_j^\dagger - \frac{1}{2}\left(L_j^\dagger L_j \rho(t) + \rho(t) L_j^\dagger L_j\right)$$

$$\begin{cases} H = -J\sum_j \left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) + \frac{U}{2}\sum_j n_j(n_j-1) \\ L_j = \sqrt{\gamma}\, b_j^2 \end{cases}$$

Typical number for Yb in a very deep optical lattice (15 E_r)

- $U/\hbar = 9500\ s^{-1}$
- $\gamma = 7000\ s^{-1}$
- $J/\hbar = 79\ s^{-1}$

Time-scales of the problem

$\frac{1}{\gamma}$ Decay time of double occupancies or more

$\frac{\hbar}{J}$ Typical time of the hopping dynamics

Idea: use this separation of timescales to develop an effective perturbative approach

Garcia-Ripoll et al., NJP (2009)

Effective master equation

Perturbative parameters: $\frac{J}{U}$ and $\frac{J}{\hbar\gamma}$

- Derivation of an effective master equation restricted to the steady space (no double occupancies or more)
- Introduce spin operators

$$\mathcal{L}'[\rho] = -\frac{i}{\hbar}[H_1, \rho] + \sum_j \left[C_j \rho C_j^\dagger - \frac{1}{2}\left(C_j^\dagger C_j \rho + \rho C_j^\dagger C_j \right) \right]$$

$$\begin{cases} H_1 = -J \sum_j \sigma_j^+ \sigma_{j+1}^- + \sigma_{j+1}^+ \sigma_j^- \\ C_j = \sqrt{\Gamma_{\text{eff}}}\, \sigma_j^- \left(\sigma_{j+1}^- + \sigma_{j-1}^- \right) \end{cases}$$

New emergent time scale

$$\Gamma_{\text{eff}} = \frac{J^2}{\hbar\gamma} \frac{2}{1 + \left(\frac{U}{\hbar\gamma}\right)^2}$$

Time-scales of the problem

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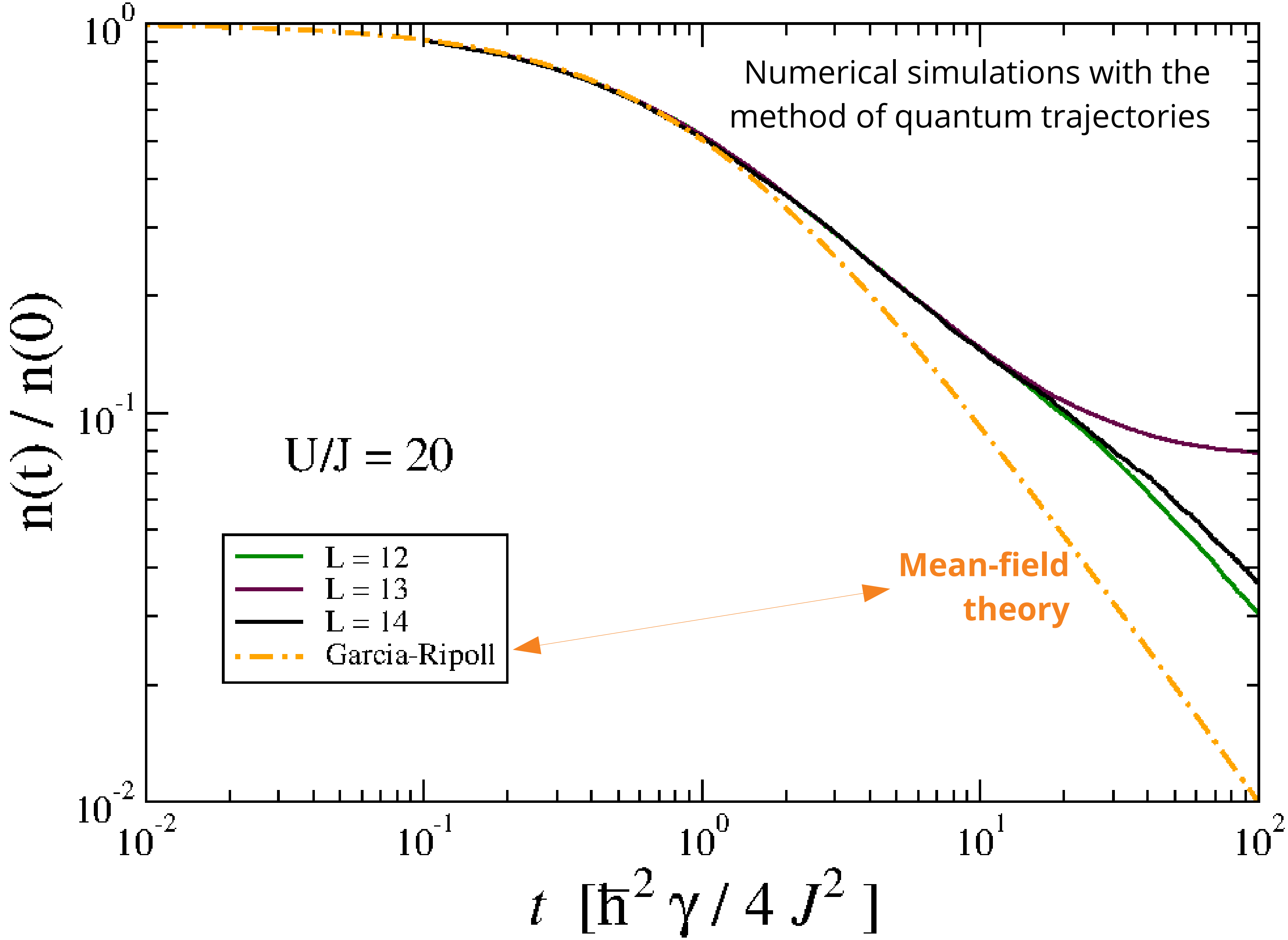
$\frac{1}{\Gamma_{\text{eff}}}$ Decay time of single occupied sites

Garcia-Ripoll et al., NJP (2009)

Numerical solution



Garcia-Ripoll et al., NJP (2009)

Our idea

Time-scales of the problem

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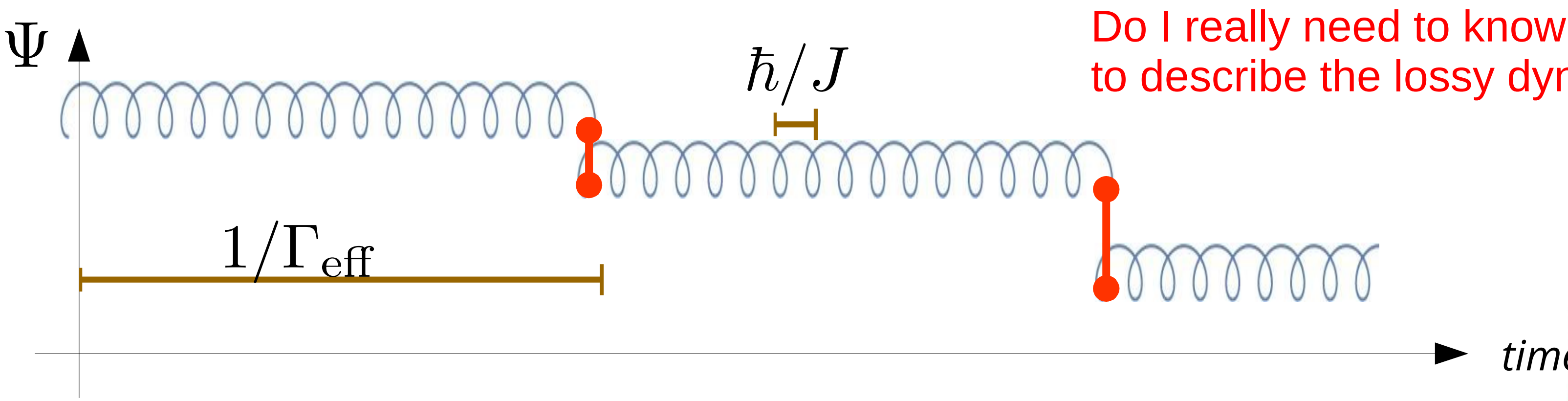
Intuitive picture: lossy events are rare.
Between two lossy events the system has a lot of time to evolve unitarily

Sketch without any mathematical meaning



Do I really need to know exactly to describe the lossy dynamics?

Generalized Gibbs ensemble

How to characterize the state $|\Psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}_1 t}|\Psi(0)\rangle$ at long times?

$$H_1 = -J\sum_j \sigma_j^+\sigma_{j+1}^- + \sigma_{j+1}^+\sigma_j^-$$

Jordan-Wigner transformation

$$H_1 = -J\sum_j c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j$$
$$= -2J\sum_k \cos(k) c_k^\dagger c_k$$

Thermalization in closed quantum systems:

- At long times the quantum state is indistinguishable from a generalised Gibbs state

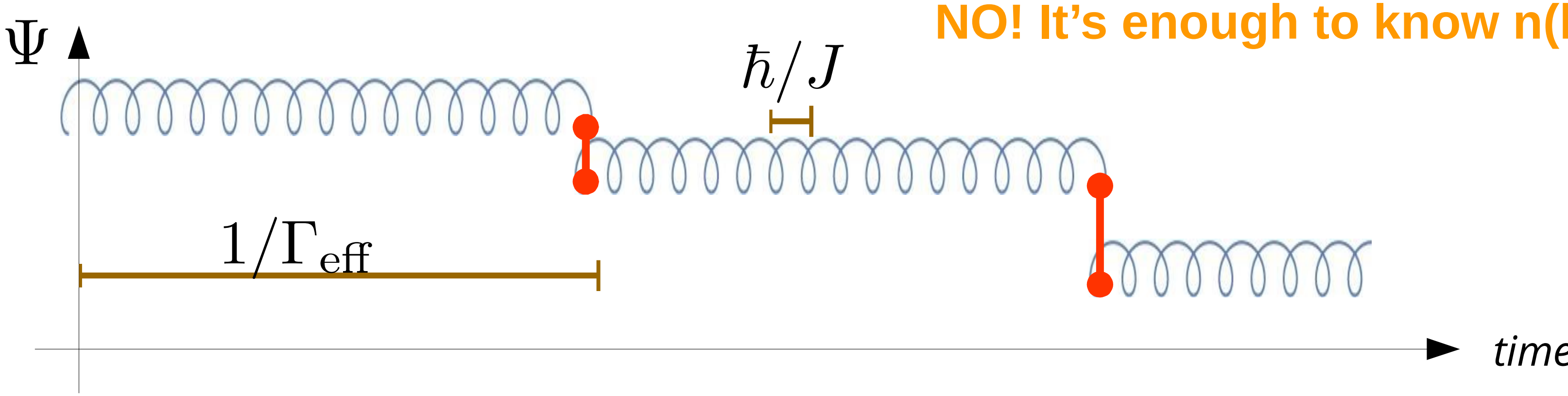
$$\lim_{t\to\infty}\langle\Psi(t)|A|\Psi(t)\rangle = \frac{1}{Z'}\mathrm{tr}\left[e^{-\sum_k \mu_k c_k^\dagger c_k} A\right]$$

Important: the coefficients μ_k are in 1-to-1 correspondence with the n(k)

Generalized Gibbs ensemble

Do I really need to know exactly $\Psi(t)$ to describe the lossy dynamics?

NO! It's enough to know n(k,t)



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See also: Lenarcic, Lange, Rosch, PRB 2017 and Mallayya, Rigol, De Roeck PRX 2019 and Bouchoule, Doyon Dubail, Scipost 2020.

Rate Equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle n_k\rangle_t = -\frac{i}{\hbar}\langle n_k\,[H_2,\rho(t)]\rangle_t + \sum_j \langle n_k L_j \rho(t) L_j^\dagger\rangle_t - \frac{1}{2}\langle n_k\{L_j^\dagger L_j,\rho(t)\}\rangle_t$$

We assume $\rho(t) = \frac{1}{Z'}\mathrm{tr}\left[e^{-\sum_k \mu_k(t) c_k^\dagger c_k}\right]$

which satisfies Wick's theorem in k space and is diagonal in l space

$$\langle c_k^\dagger c_q^\dagger c_s c_z\rangle_t = (\delta_{kz}\delta_{qs} - \delta_{ks}\delta_{qz})\, n_k(t) n_q(t)$$

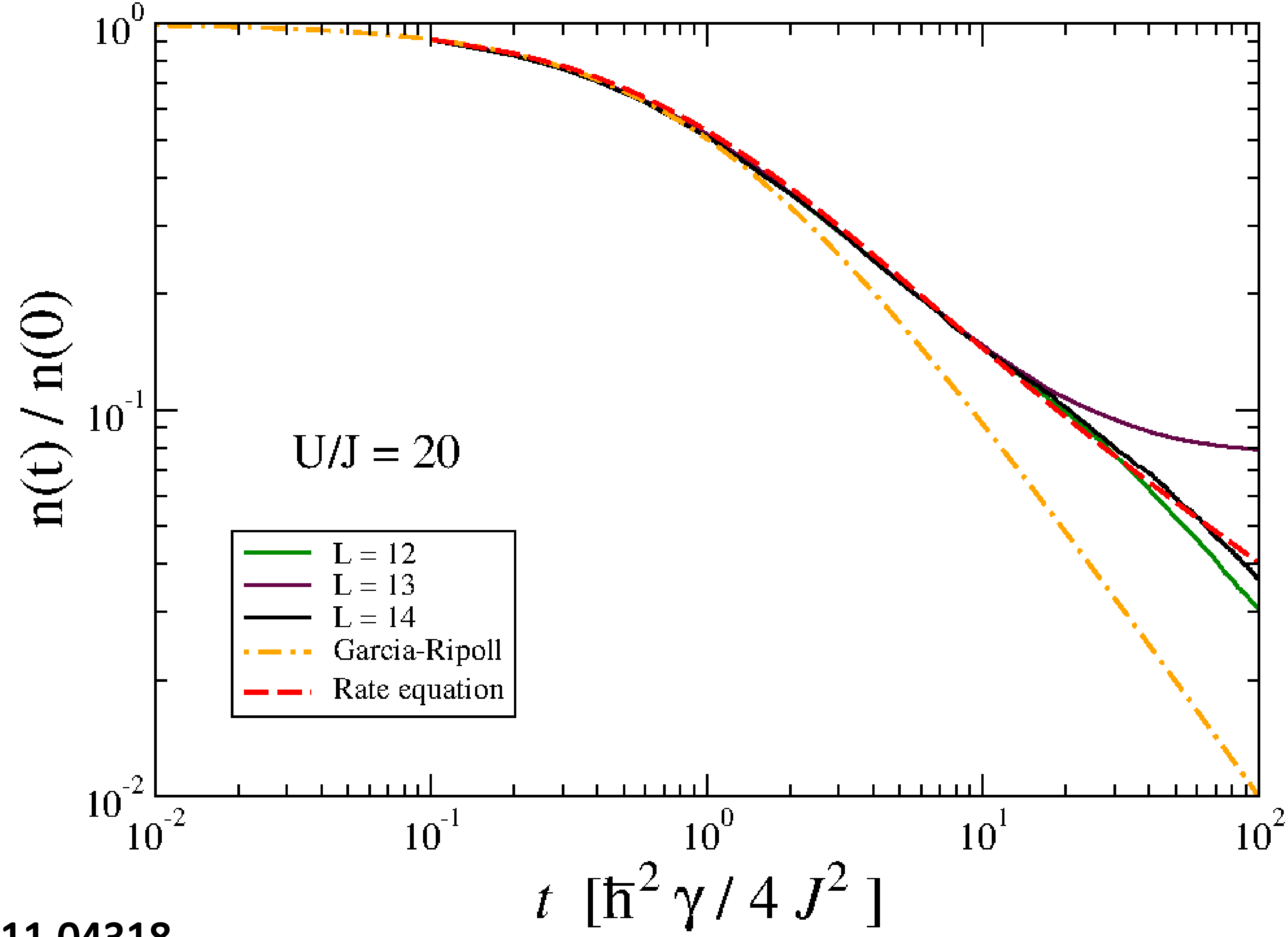
Time-dependent generalized Gibbs ensemble

$$\frac{\mathrm{d}}{\mathrm{d}t} n_k(t) = -\frac{4\Gamma_{\mathrm{eff}}}{L}\sum_q n_q(t) n_k(t)[\sin(k) - \sin(q)\,]^2$$

Huge simplification of the problem.

See also: Lenarcic, Lange, Rosch, PRB 2017 and Mallayya, Rigol, De Roeck PRX 2019 and Bouchoule, Doyon Dubail, Scipost 2020.

Numerical results - again



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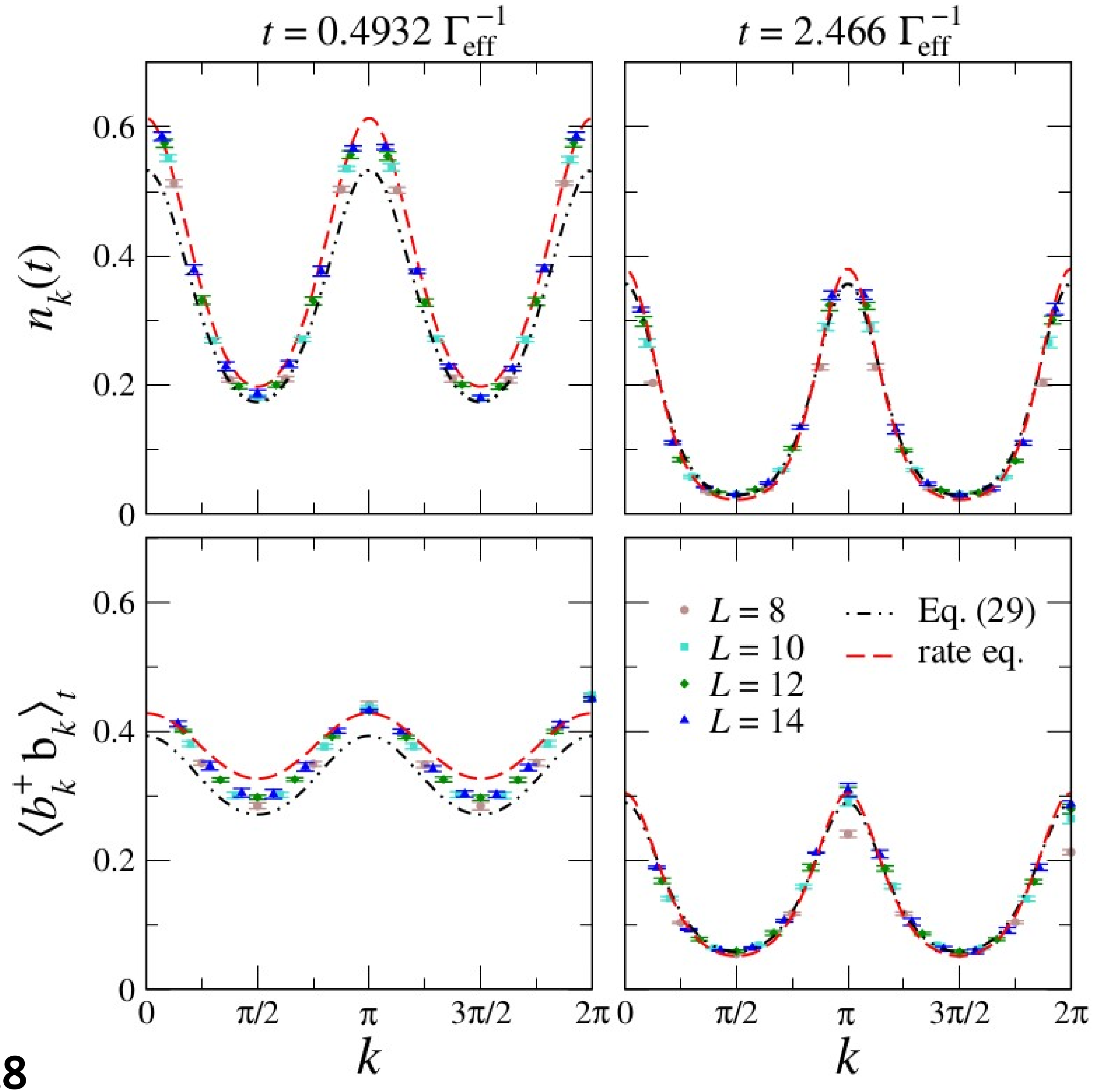
Properties of our solution



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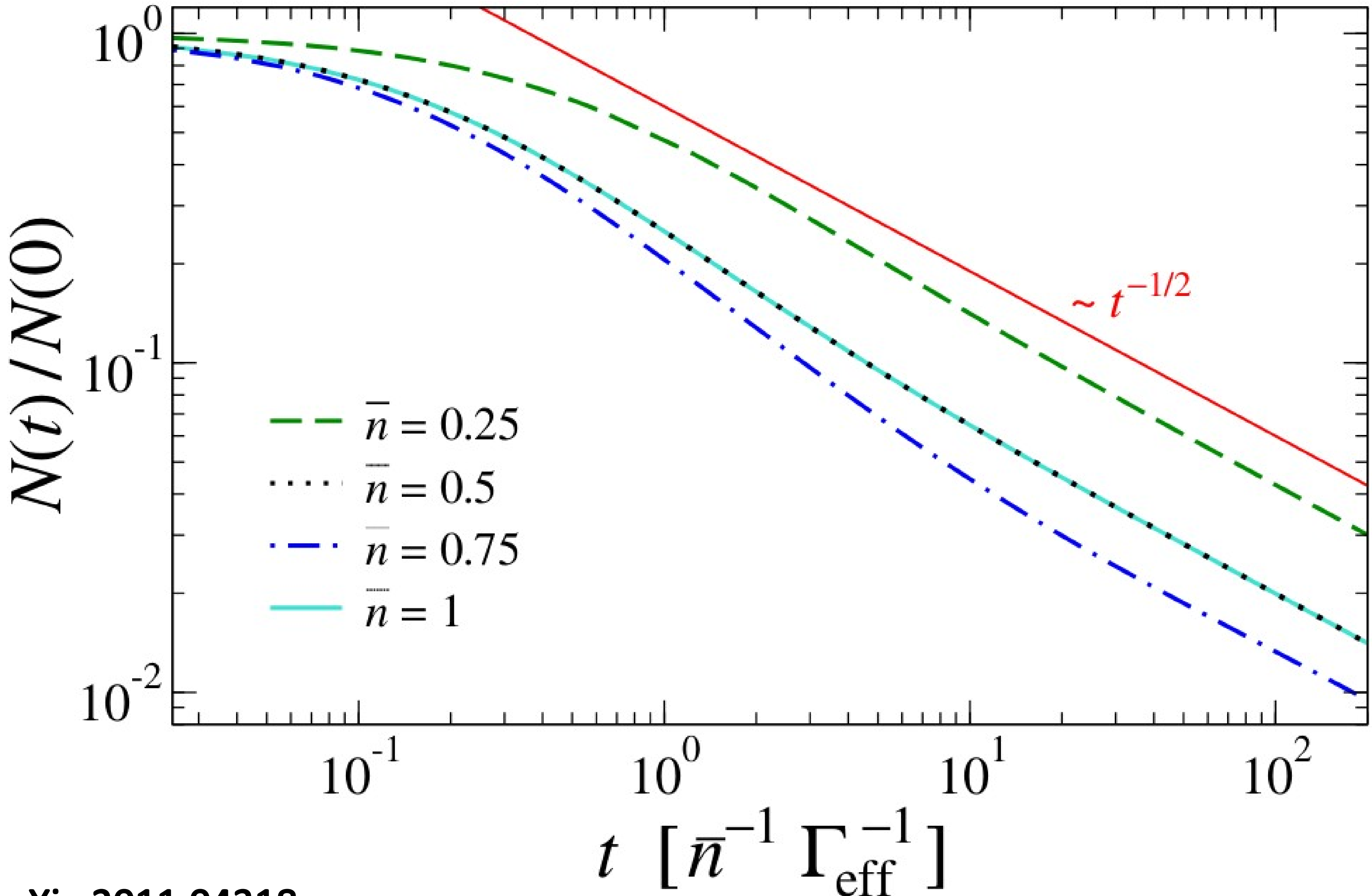

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