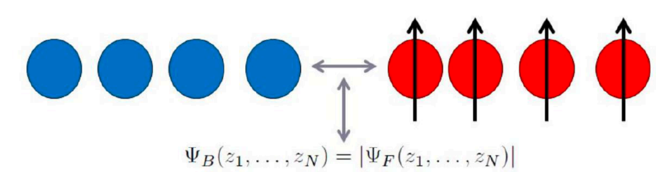
Thermodynamics of a 1D Bose gas with repulsive contact interactions - G. De Rosi

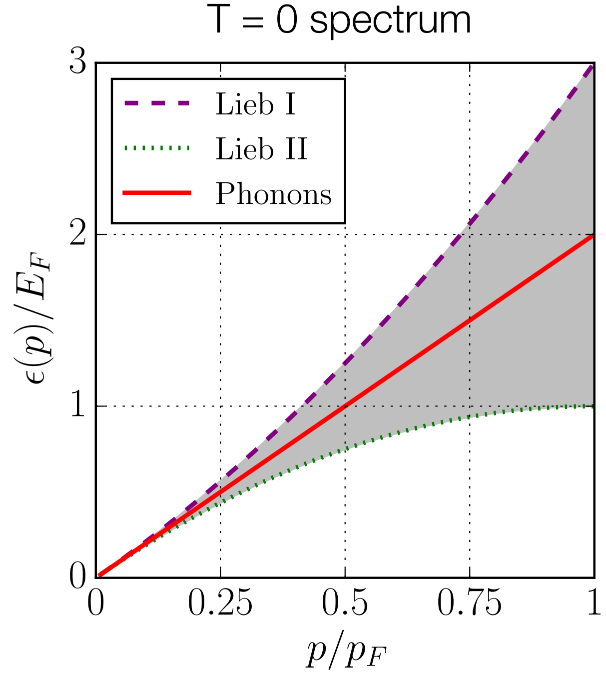
$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g\sum_{i>j}^{N}\delta(x_i - x_j)$$

$g = -2\hbar^2/(ma) > 0$	Coupling constant
$a < 0$	Scattering length
$\gamma = -2/(na)$	Interaction strength
$n = N/L$	Density

- $\gamma \ll 1, \quad n|a| \gg 1$ (weak repulsion): Gross-Pitaevskii (GP)
- $\gamma \to \infty, \quad n|a| \to 0$ (strong repulsion): Tonks-Girardeau (TG) ⟶ Ideal Fermi Gas (IFG)



T = 0 spectrum



⟶ Bogoliubov spectrum (GP)
Particle IFG spectrum (TG)

Continuous structure at larger momenta

⟶ Dark soliton spectrum (GP)
Hole IFG spectrum (TG)

⟶ Higher-order low-T thermodynamic corrections

Lieb and Liniger (1963)
Lieb (1963)

Analytical limits for the chemical potential

Weak repulsions & low T: Bogoliubov (BG)

Ideal gas of bosonic quasi-particles

$$\epsilon(p) \approx v_s(\gamma)\,|p|\left[1 + \frac{p^2}{8m^2 v_s^2(\gamma)}\right]$$

Luttinger Liquid term $O(T^2)$ for any interaction strength γ

Beyond Luttinger Liquid term $O(T^4)$ (only small γ)

$$\Delta\mu_{\rm BG} \approx \frac{\pi}{6}\frac{(k_B T)^2}{\hbar v_s^2}\left(\frac{\partial v_s}{\partial n}\right)_L\left[1 - \frac{\pi^2}{4}\frac{(k_B T)^2}{m^2 v_s^4}\right]$$

$v_s(\gamma)$ T = 0 sound velocity

Weak repulsions & high T: Virial expansion

$$\mu_{\rm GP} \approx k_B T\left[\ln(n\lambda) - \frac{n\lambda}{\sqrt{2}}\right]$$

Thermal wavelength $\lambda = \sqrt{2\pi\hbar^2/(mk_BT)}$

Strong repulsions at any T: Hard-Core (HC)

Impenetrable bosons of diameter a

Free energy $A_{\rm HC} = A_{\rm IFG}(L \to \hat{L})$ $\quad \hat{L} = L - Na$

Low T: Sommerfeld

$$\mu_{\rm HC} \approx \hat{E}_F\left[\left(1 + \frac{2}{3}a\hat{n}\right) + \frac{\pi^2}{12}\hat{\tau}^2\left(1 + 2a\hat{n}\right) + \frac{\pi^4}{36}\hat{\tau}^4\left(1 + \frac{6}{5}a\hat{n}\right) + \frac{7\pi^6}{144}\hat{\tau}^6\left(1 + \frac{10}{9}a\hat{n}\right)\right]$$

$$\hat{\tau} = \frac{k_B T}{\hat{E}_F}$$

High T: Virial

$$\mu_{\rm HC} \approx k_B T\left[\ln\left(\hat{n}\lambda\right) + a\hat{n} + \left(1 + \frac{a\hat{n}}{2}\right)\frac{\hat{n}\lambda}{\sqrt{2}}\right]$$

Effective Fermi energy $\hat{E}_F = \dfrac{\hbar^2\pi^2\hat{n}^2}{2m}$

$$\hat{n} = \frac{n}{1 - an}$$

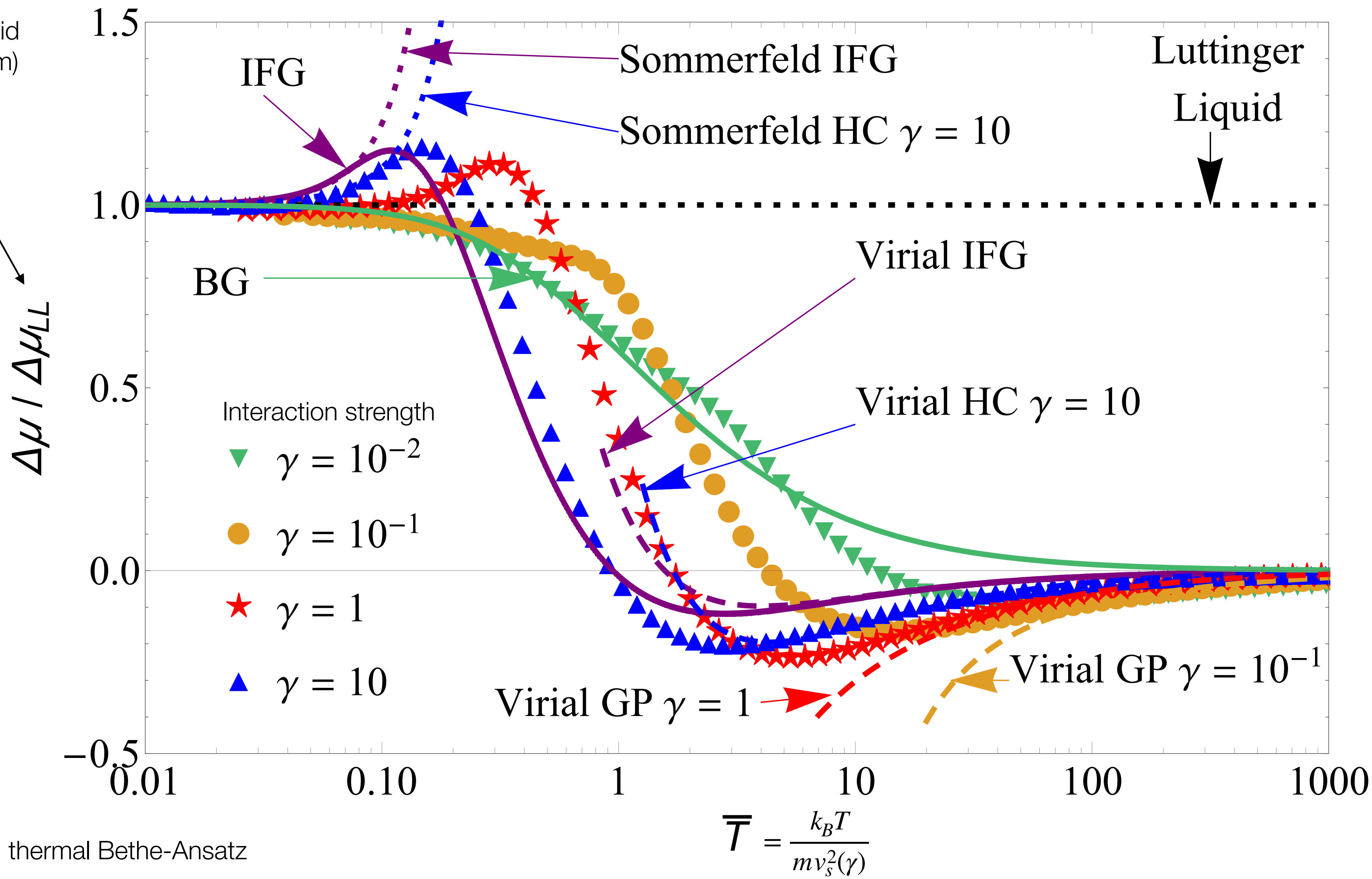
De Rosi, Astrakharchik and Stringari (2017)
De Rosi, Massignan, Lewenstein and Astrakharchik (2019)

Thermal chemical potential as a function of T

LL = Luttinger Liquid
(phononic spectrum)

$\Delta\mu_{LL} \approx \alpha(\gamma)T^2$

$\Delta\mu = \mu(T) - \mu_0$



- Symbols = numerical thermal Bethe-Ansatz

- BG = Bogoliubov (weak repulsion)
- GP = Gross-Pitaevskii (weak repulsion)
- HC = Hard-Core (strong repulsion)
- IFG = Ideal Fermi Gas (TG)

De Rosi, Astrakharchik and Stringari (2017)
De Rosi, Massignan, Lewenstein and Astrakharchik (2019)

Conclusions

- Pressure
- Tan's contact $\Delta\mathscr{C}_{\mathrm{BG}} \propto \Delta\mu_{\mathrm{BG}}$ $\mathscr{C}_{\mathrm{HC}} = \frac{4mN}{\hbar^2}P_{\mathrm{HC}}$

De Rosi, Massignan, Lewenstein and Astrakharchik (2019)
Barth and Zwerger (2011)

- Experimental observability of all thermodynamic quantities

Salces-Carcoba et Al. (2018)
Wild et Al. (2012)
Sagi et Al. (2012)
Yan et Al. (2019)
Hoinka et Al. (2013)
Stewart et Al. (2010)
Chang et Al. (2016)

- Numerical thermal Bethe-Ansatz VS analytical limits
- Microscopic explanation of beyond-Luttinger-Liquid next-to-leading low-T corrections:
  - Non-linear BG spectrum for weak repulsion (negative sign)
  - HC model (volume excluded effects) for strong repulsion (positive sign)

Outlooks

- Impurities in quantum bath
- Breathing modes in harmonic trap
- Multicomponent systems
- Finite number of atoms
- 1D Bose-Bose liquid droplets
- Super Tonks-Girardeau
- Finite-range interactions (Dipolar, Rydberg & Helium systems)

Contact me!

giulia.derosi88@gmail.com
https://giuliaderosi.wordpress.com/

De Rosi, Astrakharchik and Stringari, Phys. Rev. A 96, 013613 (2017)
De Rosi, Massignan, Lewenstein and Astrakharchik, Phys. Rev. Research 1, 033083 (2019)