Loschmidt echo singularities as dynamical signatures of strongly localized phases

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Problem and objective

Much of the phenomenology of MBL dynamics can be directly explained in terms of the existence of l-bits and interactions between them[1]: yet observing l-bits directly is an arduous task, given that their expression is highly disorder-dependent (and generally unknown even in theory), and it would require high-precision measurements of local observables in different local bases. The purpose of this work is to show that, in the case of strongly localized regimes, the existence of l-bits can offer striking signatures in the dynamics of the Loschmidt echo (LE), namely in the logarithm of the return probability to the initial state $|\psi_0\rangle$

$$\lambda(t) = -\frac{1}{L}\left[\log|\langle\psi_0|e^{-i\mathcal{H}t}|\psi_0\rangle|^2\right]_{\mathrm{av}} .$$

Models and protocol

Spinless fermions with nearest-neighbor interactions in an inhomogeneous local potential,

$$\mathcal{H} = \sum_{i=1}^{L-1}\left[-\frac{J}{2}\left(c_i^\dagger c_{i+1} + \mathrm{h.c.}\right) + J_z n_i n_j\right] - \sum_{i=1}^{L} h_i n_i$$

- h_i is taken to be either quasi-periodic (QP), namely $h_i = \Delta\cos(2\pi\kappa i + \phi)$ with $\kappa = 0.721$, and ϕ a random phase; or to be fully random (FR) and uniformly distributed in the interval $[-\Delta, \Delta]$.
- Chains of length $L = 22$ with open boundaries, and average over $\sim 10^3$ realizations of the random phase (QP) or of the full random potential (FR)
- Exact diagonalization (ED) simulations of quench dynamics starting from $|\psi_0\rangle = |1010101...\rangle$

Conclusions and references

- Sharp cusp-like singularities in the Loschmidt echo (LE) are a generic feature of the localized dynamics of an extended quantum system initialized in a factorized state.
- These features can be fully explained by the dynamics of an ensemble of effective two-level (or even three-level) systems, offering an explicit approximation to the conserved l-bits in the MBL regime.
- The faster decay in the dynamics of the LE compared to that predicted by the 2LS/3LS models is a direct manifestation of the interactions between the l-bits, namely the defining feature of many-body localization (MBL)

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[3] M.O.Scully, M. Zubairy: Quantum Optics, *Cambridge Univ. Press*, (1997)

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LE singularities and imbalance oscillations

- Dynamics of the LE, $\lambda(t)$, along with that of the imbalance $I(t) = \sum_i(-1)(2[\langle n_i\rangle]_{\mathrm{av}} - 1)/L$ (which saturates to its maximum value of 1 in the initial state and probes the persistence of the initial density/spin pattern).
- We observe that for both the QP and FR potentials, for disorder strengths compatible with the onset of the MBL regime the LE displays a sequence of periodic cusp-like peaks at times $t_n = (2n+1)\pi/J$ $(n = 0, 1, 2, ...)$, which correspond to minima of the imbalance.
- These cusps represent a strong signature of MBL-type behavior in the *short-time dynamics*, experimentally accessible [2].

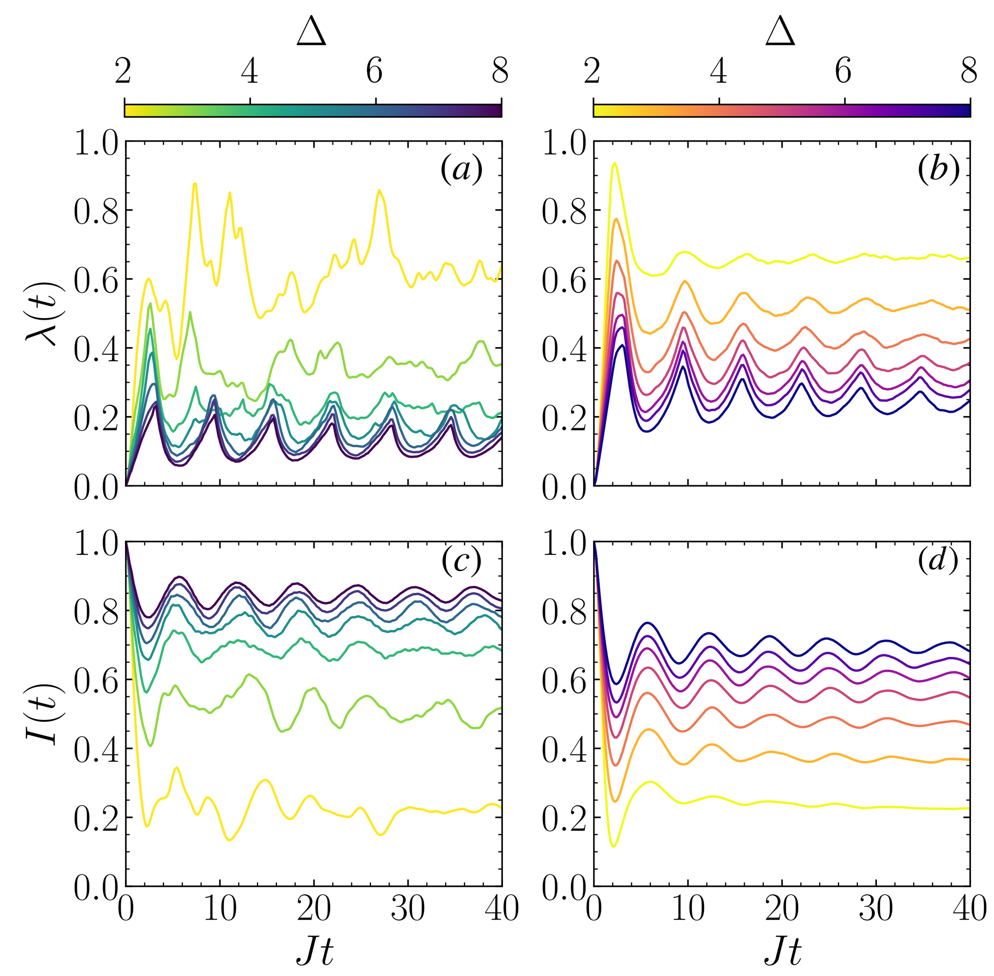

Figure 1: *Loschmidt echo and imbalance dynamics for QP potential (a-c) and FR potential (b-d), for various disorder strengths Δ.*

2LS model

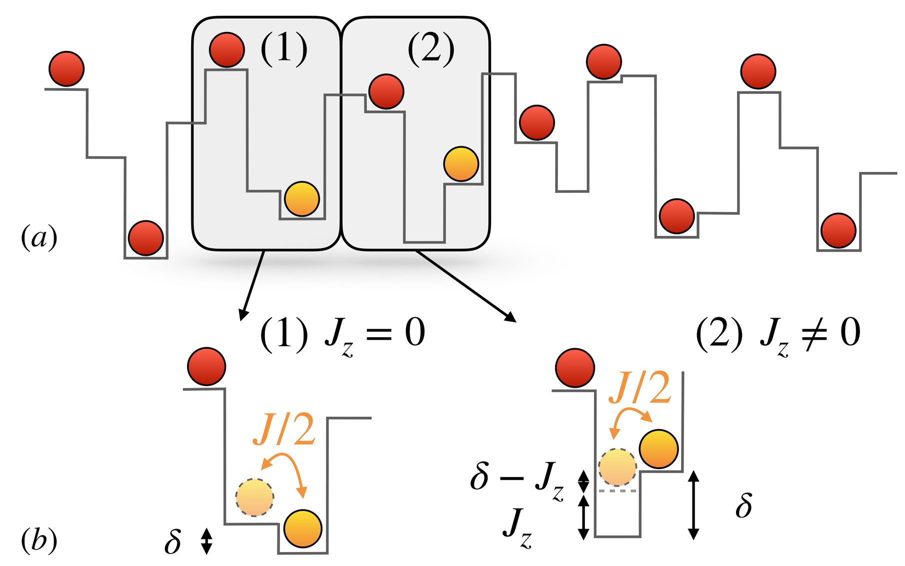

Figure 2: *(a) Example of a $L = 22$ chain in a QP potential in the initial CDW state $|1010...\rangle$. (b) Zoom on two quasi-resonant regions: in the case of $J_z = 0$ the region (1) presents a pair of quasi-resonant sites for the particle in orange; in the case of $J_z \neq 0$, region (2) shows two quasi-resonant sites for the orange particle, thanks to the partial screening of disorder offered by the interaction with the red particle.*

- Collective dynamics of an ensemble of independent two-level systems (2LS) detuned by δ (explicit approximation of the conserved l-bits).
- For N two-level systems with local detunings δ_n, oscillating with a Rabi frequency Ω, the LE is
$$\lambda(t) = -\frac{1}{L}\sum_n \log\left[1 - p(\delta_n', J, t)\right],$$
where $p(\delta, \Omega, t)$ is the probability of finding the 2LS in the state orthogonal to the initial one [3].
- When averaging over disorder, it is immediate to obtain the following simple expression
$$\lambda(t) = -\int P(\delta' + J_z)\log\left[1 - p(\delta', J, t)\right].$$

From 2LS to 3LS and the long-time dynamics

To take "rare" regions in which contiguous pairs of sites ($(i-1, i)$ and $(i, i+1)$) are nearly resonant at the same time, we extend the model to include clusters of 3 sites, i.e. obtaining collective dynamics of an ensemble of three-level systems (3LS). Beside the LE, the 2LS and 3LS models offer the opportunity of obtaining simple analytical expression for other observables, e.g. the imbalance $I(t)$ and the entanglement entropy [4].

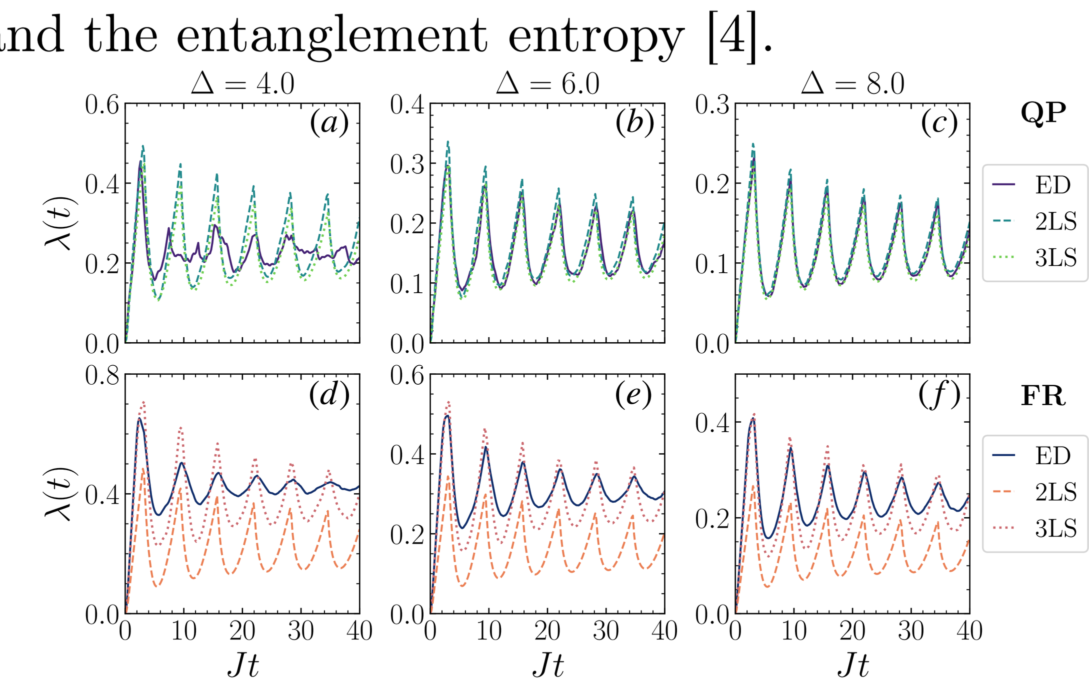

Figure 3: *Comparison between the LE $\lambda(t)$ and the predictions of the 2LS and 3LS models: (a-c) QP potential; (d-f) FR potential.*

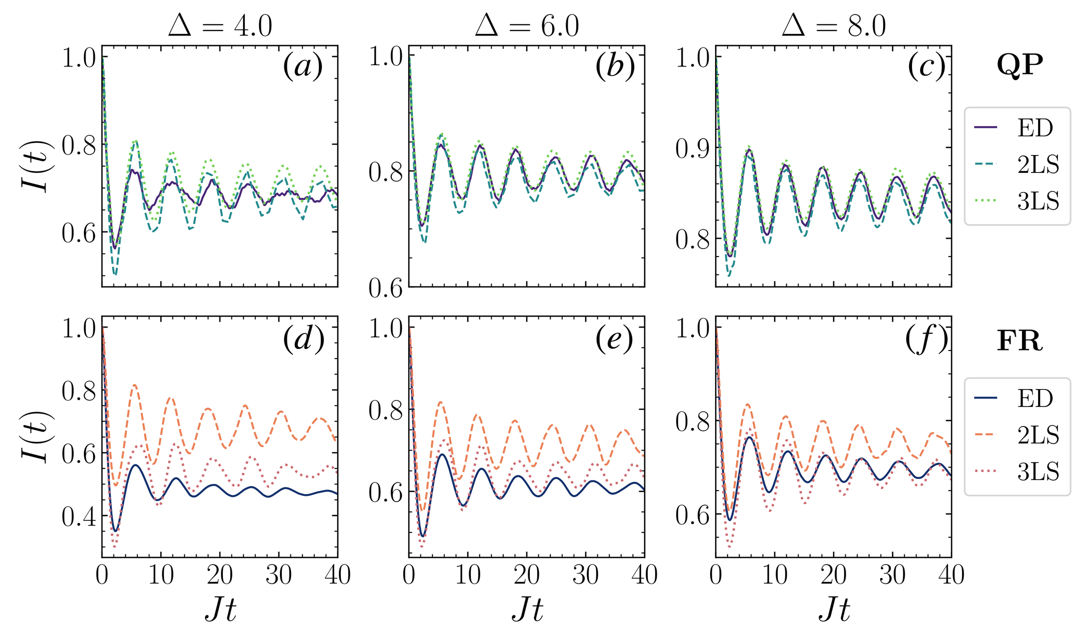

Figure 4: *Comparison between the imbalance $I(t)$ and the predictions of the 2LS and 3LS models: (a-c) QP potential; (d-f) FR potential.*

- The cusp height decays in time as $\sim t_n^{-1/2}$ for the 2LS and 3LS models.
- ED data for MBL systems are found to display a strong deviation from the 2LS model prediction, exhibiting a much faster decay.
- This crossover is the manifestation of l-bit interactions, which are a defining feature of MBL and lead to interaction-induced dephasing (IID).

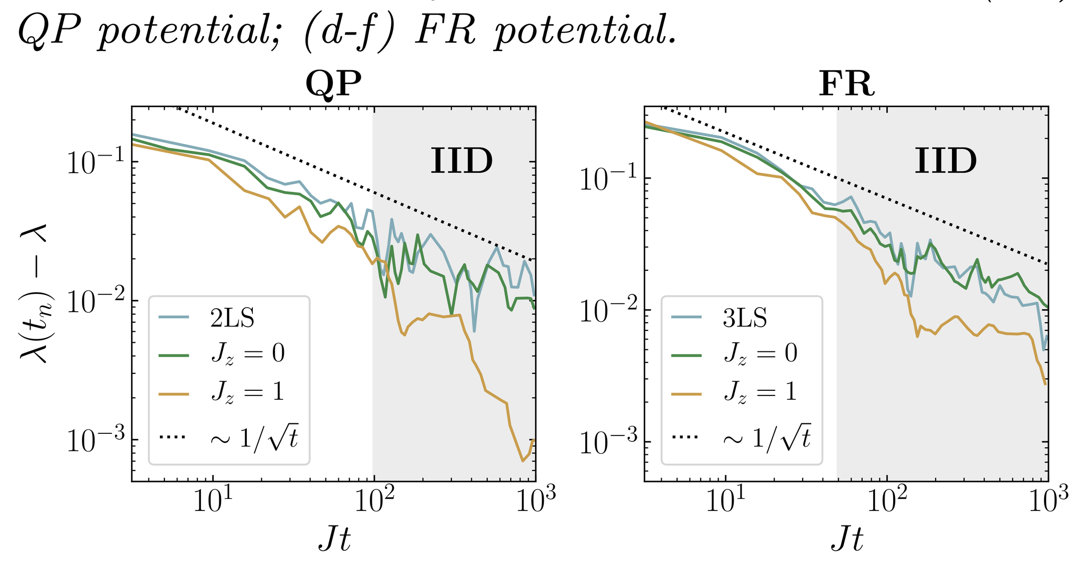

Figure 5: *Decay of the peaks of $\lambda(t)$, $\max_t \lambda(t) - \bar{\lambda}$. Left: QP potential; Right: FR potential.*