Vortex Reconnections across the BCS-BEC Crossover

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1. Density Functional Theory for Fermi Superfluids

✓ W Zwerger, Ed., *The BCS-BEC crossover and the Unitary Fermi Gas*, 2012.
✓ Bulgac, Yu., PRL 88, 2002.

Bogoliubov-de Gennes equations

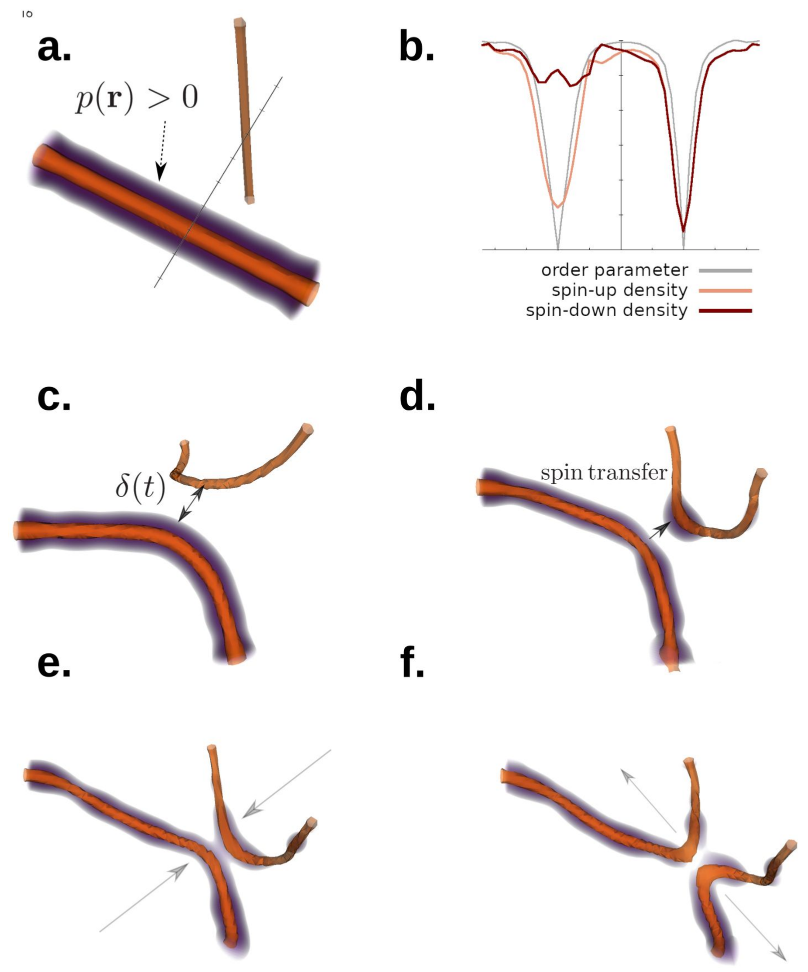
1. The energy functional of the mean-field superfluid Fermi gas in the BCS regime is $E = \int d\mathbf{x}\, \mathcal{E}(\mathbf{x}), \quad \mathcal{E}(\mathbf{x}) = \hbar^2(\tau_\uparrow + \tau_\downarrow)/2m + |\Delta|^2/g$
$$\tau_\uparrow = \sum_\eta |\nabla u_\eta|^2 f(E_\eta), \quad \tau_\downarrow = \sum_\eta |\nabla v_\eta|^2 f(-E_\eta), \quad \Delta(\mathbf{x}) = -g_{\text{eff}} \sum_\eta u_\eta(\mathbf{x}) v^*_\eta(\mathbf{x})$$
2. The energies (E_η) and wave functions of quasiparticles $(u_\eta, v_\eta)^T$ are obtained as self-consistent solutions to the Bogoliubov-de Gennes equations
$$\begin{pmatrix} \mathcal{H}_{\text{BdG}} - \mu_\uparrow & \Delta(\mathbf{x}) \\ \Delta^\dagger(\mathbf{x}) & -\mathcal{H}_{\text{BdG}} + \mu_\downarrow \end{pmatrix} \begin{pmatrix} u_\eta(\mathbf{x}) \\ v_\eta(\mathbf{x}) \end{pmatrix} = E_\eta \begin{pmatrix} u_\eta(\mathbf{x}) \\ v_\eta(\mathbf{x}) \end{pmatrix}$$
Here $\mathcal{H}_{\text{BdG}} = -\hbar^2\nabla^2/2m$
3. The renormalization requires the energy cut-off and a renormalized g_{eff}

Asymmetric Superfluid Local Density Approximation (ASLDA)

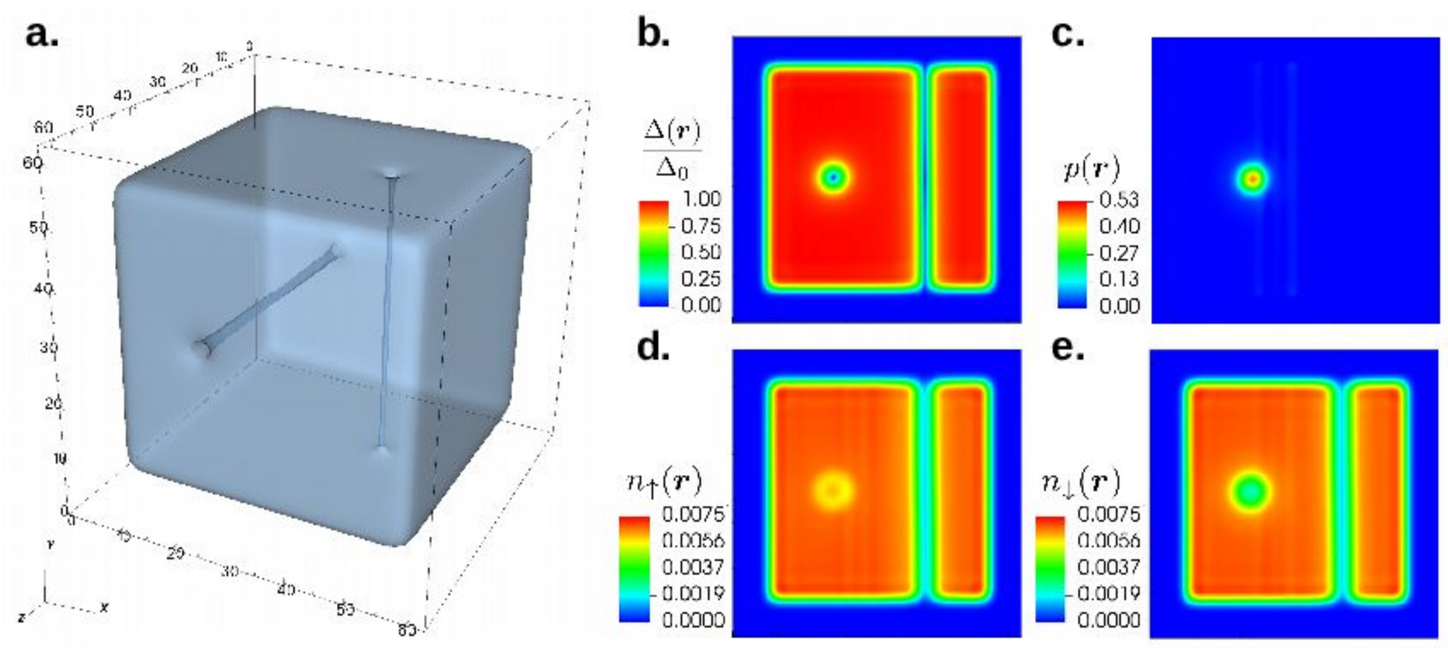
1. Extends the BdG functional to the Fermi superfluid gas at unitarity (UFG).
2. The functional reads as
$$\mathcal{E}(\mathbf{x}) = \frac{\hbar^2}{2m}\left(\alpha_\uparrow(n_\uparrow, n_\downarrow)\tau_\uparrow + \alpha_\downarrow(n_\uparrow, n_\downarrow)\tau_\downarrow + D(n_\uparrow, n_\downarrow)\right) + \frac{|\Delta|^2}{g(n_\uparrow, n_\downarrow)},$$
with parameters $\alpha_\updownarrow$ and D have to be determined ab initio (e.g. Quantum Monte Carlo simulations).
3. The minimization procedure consists of self-consistent iterations of the modified BdG equations similarly to the BCS case.
4. By replacing $E_\eta \to i\hbar\frac{\partial}{\partial t}$ we obtain time-dependent version of ASLDA equations.

2. Vortex Collisions with a spin polarization

1. Initially perpendicular vortex lines, where only one carries a spin polarization.
2. At short distances vortex lines interact due to phase configuration.
3. Before the reconnection event there is a spin polarization transfer between the approaching vortex lines.
4. The vortex lines tend to assume an antiparallel configuration before they reconnect.
5. After the reconnection the two vortex lines have a similar spin polarization.

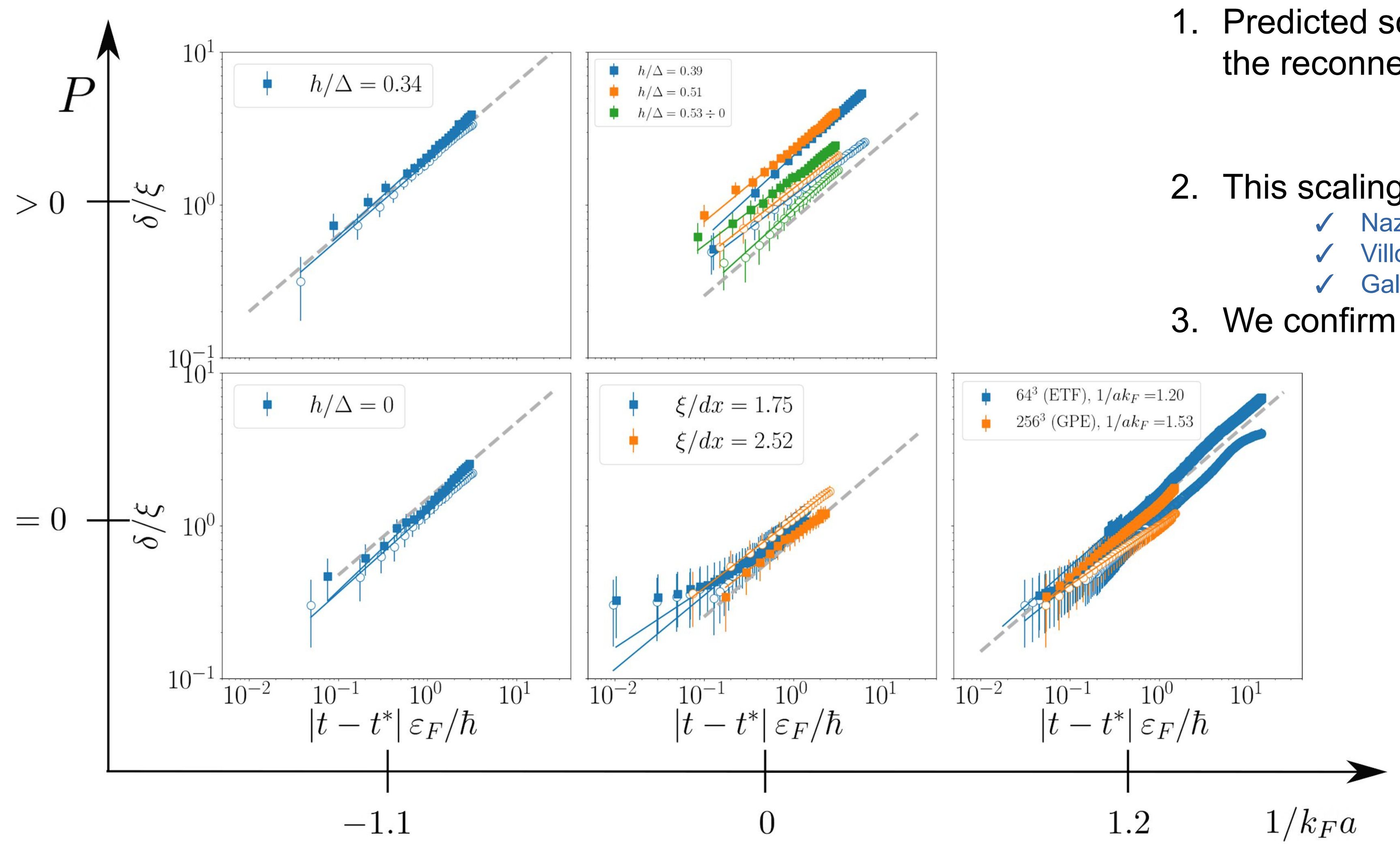


3. Initial and boundary conditions

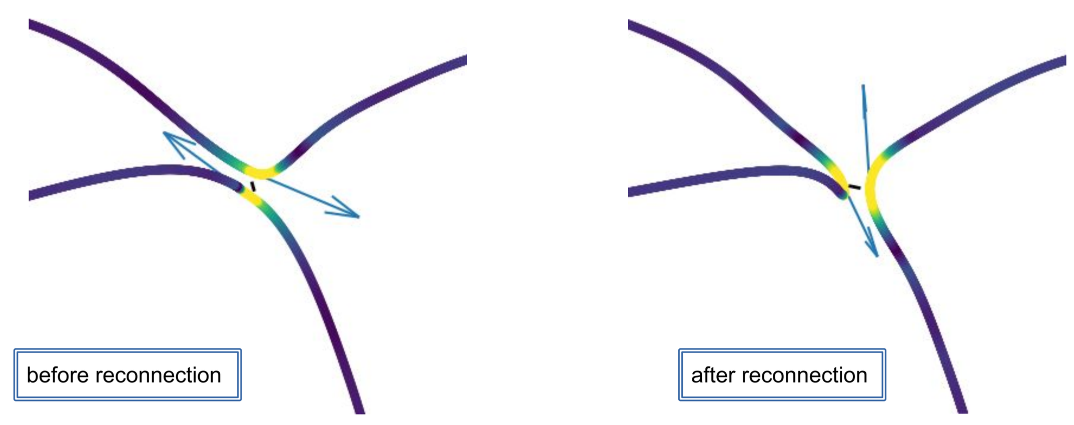


1. Initially vortex lines are perpendicular.
2. At short distances vortex lines interact mostly due to phase configuration.
3. The potential wall at the boundary is chosen such that it is smooth (vanishing all derivatives).

4. Scaling of a distance between the vortex lines



1. Predicted scaling of the distance between the colliding vortex lines vs time to the reconnection time t^* is
$$\delta \sim |t - t^*|^{1/2}.$$
2. This scaling was predicted and tested for a linearized model and for a BEC.
✓ Nazarenko and West, J. Low Temp. Phys. 2003
✓ Villois, Proment, and Krstulovic, Phys. Rev. Fluids 2, 2017
✓ Galantucci *et al.*, PNAS 2019
3. We confirm this scaling as **universal** also in the BCS regime and for the UFG.

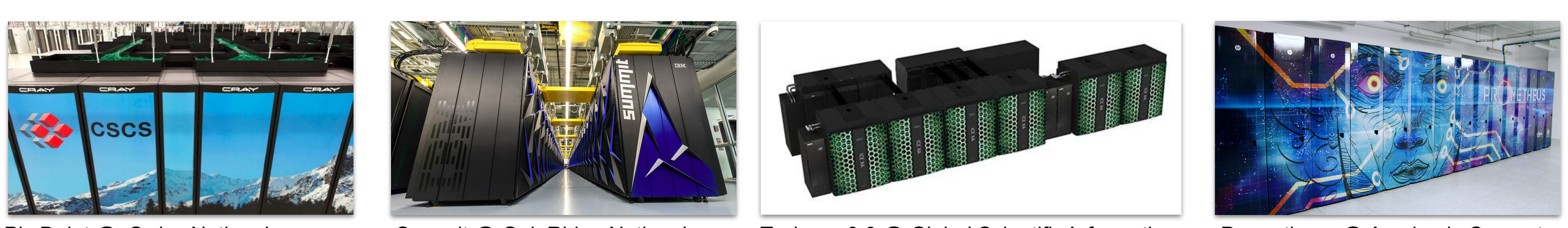


5. High Performance Computing

Simulations methods

✓ Simulations are conducted on a 3D spatial lattice of size $N_x \times N_y \times N_z$ with no symmetry restrictions.
✓ We use massively parallelized numerical codes with GPU acceleration
✓ With present supercomputers capabilities we can study 3D dynamics of systems containing up to 3×10^5 of ultracold fermions, with arbitrary spin-polarization.

Selected hpc systems in use by our group



Piz Daint @ Swiss National Supercomputing Centre (Switzerland) | Summit @ Oak Ridge National Laboratory (USA) | Tsubame3.0 @ Global Scientific Information and Computing Center, (Tokyo) | Prometheus @ Academic Computer Centre CYFRONET (Poland)

Future plans

✓ Release of our hpc codes in form open source toolkit (in 2021)
✓ Extension of the toolkit towards Bose-Fermi mixtures

Acknowledgments

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