Impurity immersed in a double Fermi sea

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Introduction

The study of dual superfluids of bosons and fermions recently paved the way to the study of a novel type of impurity systems where the impurity is immersed in a spin 1/2 fermionic superfluid. The dressing of the impurity by particle-hole excitations allows the system access to different regimes starting from a *polaron* regime where the impurity with the surrounding excitations form a quasi-particle to a *trimer* regime where the impurity binds with two fermions to form an Efimov trimer [1].
We present a variational calculation of the energy of an impurity immersed in a double Fermi sea of non-interacting Fermions. We show that in the strong-coupling regime, the system undergoes a first-order transition between polaronic and trimer states. Our result suggests that the smooth crossover predicted in previous literature [2] for a superfluid background is the consequence of Cooper pairing and is absent in a normal system [3].

General Framwork

The Hamiltonian of the considered system takes the general form

$$\hat{H} = \sum_{\boldsymbol{k},s} \frac{\hbar^2 k^2}{2m} \hat{a}^\dagger_{\boldsymbol{k},s}\hat{a}_{\boldsymbol{k},s} + \sum_{\boldsymbol{k}} \frac{\hbar^2 k^2}{2m_i} \hat{b}^\dagger_{\boldsymbol{k}}\hat{b}_{\boldsymbol{k}} + \sum_{\boldsymbol{k}} \left(\frac{\hbar^2 k^2}{2M} + E_0\right) \hat{c}^\dagger_{\boldsymbol{k},s}\hat{c}_{\boldsymbol{k},s} + \frac{\Lambda}{\sqrt{\Omega}} \sum_{\boldsymbol{k},\boldsymbol{k}'} \left[\hat{a}_{\boldsymbol{k}s}\hat{b}_{\boldsymbol{k}'}\hat{c}^\dagger_{\boldsymbol{k}+\boldsymbol{k}',s} + \text{h.c.}\right], \quad (1)$$

We search for the ground state energy within a variational space spanned by the states depicted in the following:

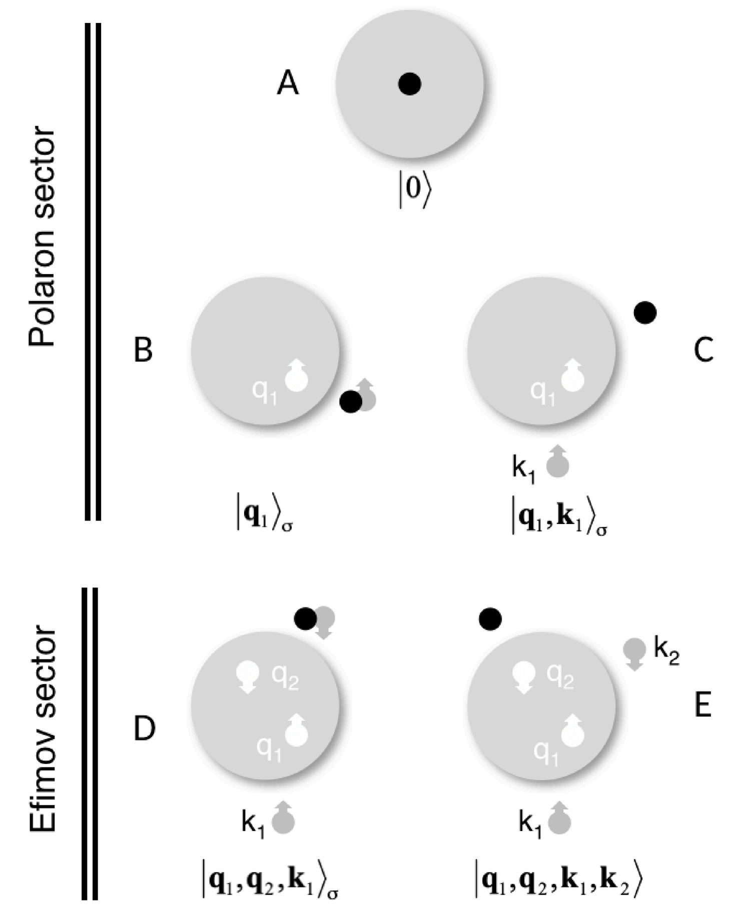

Figure 1. Structure of the variational Hilbert space.

The general structure of a variational state is therefore

$$|\psi\rangle = A|0\rangle + \sum_{\boldsymbol{q}_1,s} B_s(\boldsymbol{q}_1)|\boldsymbol{q}_1\rangle_s + \sum_{\boldsymbol{q}_1,\boldsymbol{k}_1,s} C_s(\boldsymbol{q}_1,\boldsymbol{k}_1)|\boldsymbol{q}_1,\boldsymbol{k}_1\rangle_s + \sum_{\boldsymbol{q}_1,\boldsymbol{q}_2,s} D_s(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{k}_1)|\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{k}_1\rangle_s + \sum_{\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{k}_1,\boldsymbol{k}_2} E(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{k}_1,\boldsymbol{k}_2)|\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{k}_1,\boldsymbol{k}_2\rangle. \quad (2)$$

We consider trimer amplitudes D_s and E of the form:

$$D_s(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{k}_1) = F(\boldsymbol{q}_1,\boldsymbol{q}_2)\tilde{D}(\boldsymbol{k}_1) \quad (3)$$
$$E(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{k}_1,\boldsymbol{k}_2) = F(\boldsymbol{q}_1,\boldsymbol{q}_2)\tilde{E}(\boldsymbol{k}_1,\boldsymbol{k}_2), \quad (4)$$

with the following normalization for the function $F(\boldsymbol{q}_1,\boldsymbol{q}_2)$: $\sum_{q_1,q_2}|F(\boldsymbol{q}_1,\boldsymbol{q}_2)|^2 = N_F^2$.

Polaron-Trimer Coupling

We consider the trimer sector with $A = B_s = C_s = 0$. In a first approach we simply assume that the role of the Fermi sea is to prevent the fermions above the Fermi surface from occupying states below k_F by taking $F(\boldsymbol{q}_1,\boldsymbol{q}_2) = \delta_{\boldsymbol{q}_1,-\boldsymbol{q}_2}\widetilde{F}(q_1)$, where $\widetilde{F}$ is peaked near the Fermi surface.
This is very similar to the celebrated Cooper pairing problem for pairs of fermions in superconductors and we observe that like for traditional Cooper pairing the presence of the Fermi sea stabilizes the trimer.
To explore a possible polaron-trimeron crossover we consider a trial wave-function $F(\boldsymbol{q}_1,\boldsymbol{q}_2) = F_0 e^{-\boldsymbol{q}_1\cdot\boldsymbol{q}_2/2\sigma^2}$, where F_0 is a normalization constant. Just like for the Cooper-like trimer, this amplitude is maximum when $\boldsymbol{q}_1 + \boldsymbol{q}_2 = 0$ and when both momenta are on the Fermi surface. The parameter σ allows us to tune continuously the width of the hole wave-function between a uniform distribution and the Cooper-like trimer configuration. The Cooper-like trimer corresponds to $\sigma = 0$ while the opposite limit ($\sigma = \infty$) corresponds to a uniform distribution F. To show the results we plot the following graph:

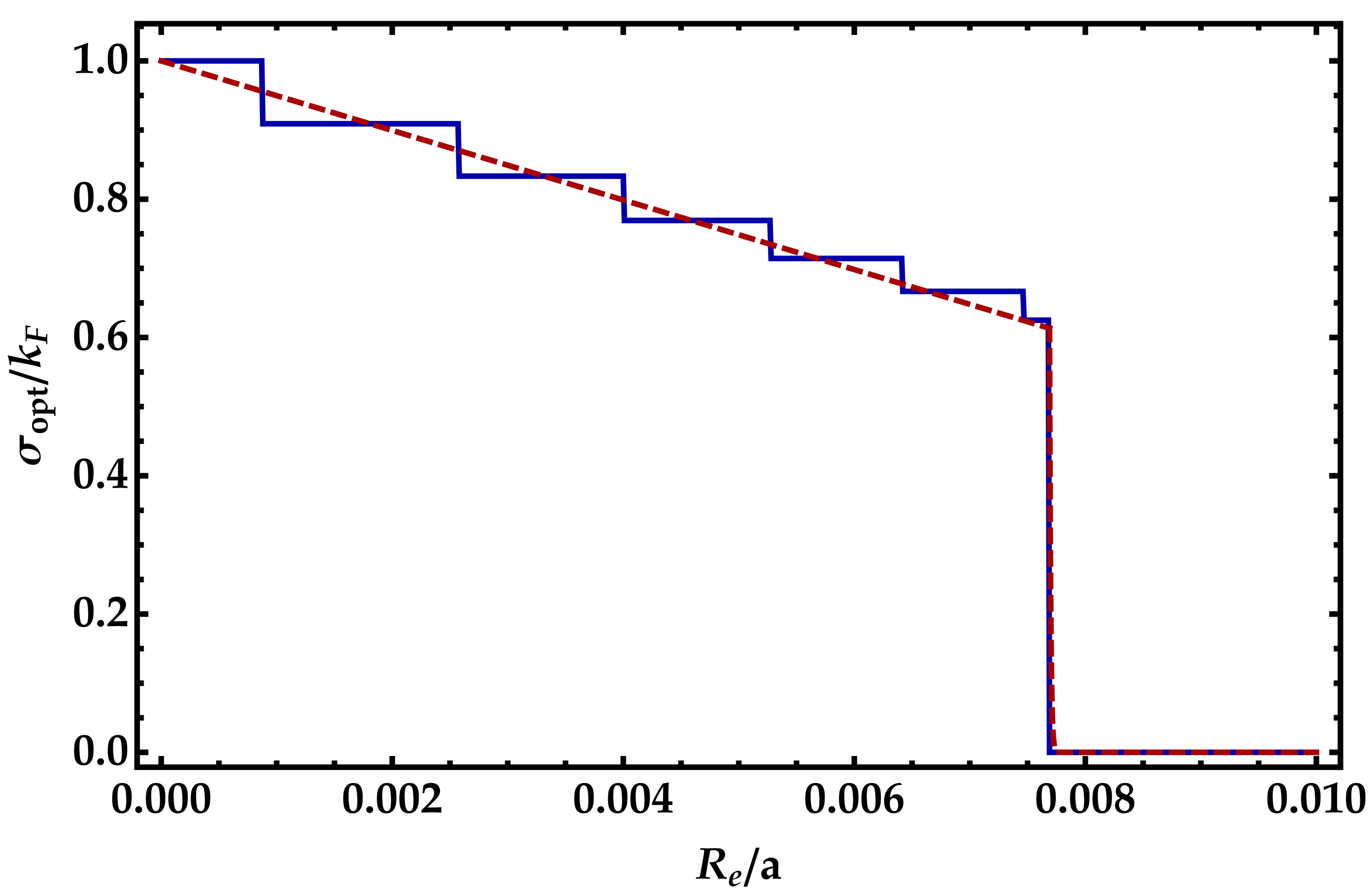

Figure 2. Optimal value of the width σ of the hole-pair wavefunction (blue solid line). σ was varied over a finite set of values between between 0 and 1 (see [3] for details).

For $R_e/a \simeq 0{,}0077$, we observe a jump of the value of σ which straddles several consecutive values of σ and thus marks a discontinuity between a Cooper-like trimer and a polaron-trimeron crossover state.

Conclusion

The absence of coupling between the two sectors comes as a result of taking the thermodynamic limit where the contribution of the holes localised at the Fermi surface to the polaron wavefunction vanish. This is different from the case of a superfluid bath, where Bogoliubov excitations help in creating a state of superposition between the impurity and many particle-hole pairs which cannot be neglected when the number of particles increase. A careful analysis of the Hilbert space might reveal the details of this behavior discrepancy.

References

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