The polarized Fermi-Hubbard superfluid at large order

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Attractive Hubbard model – 3D cubic lattice

Hamiltonian of the model

$$H = H_{\text{kin}} - \sum_{\sigma=\uparrow,\downarrow} \mu_\sigma N_\sigma + H_{\text{int}}$$

$$H_{\text{kin}} = -t \sum_{\langle \mathbf{i},\mathbf{j}\rangle\,\sigma} (c^\dagger_{\mathbf{i}\sigma} c_{\mathbf{j}\sigma} + h.c.) \qquad H_{\text{int}} = U \sum_{\mathbf{i}} n_{\mathbf{i}\uparrow}\, n_{\mathbf{i}\downarrow}$$

Fixing some parameters $t = 1\,, \quad U = -5\,, \quad \mu \equiv \frac{\mu_\uparrow + \mu_\downarrow}{2} = -3.38$

$\Rightarrow$ $\langle n_\uparrow + n_\downarrow \rangle \sim 0.5$ (quarter filling)

Diagrammatic expansion in the superfluid phase

Unperturbed quadratic Hamiltonian

$$H_0 = H_{\text{kin}} - \sum_\sigma \mu_{0,\sigma} N_\sigma + H^{(\Delta_0)}_{\text{pair}}$$

$$H^{(\Delta_0)}_{\text{pair}} \equiv \Delta_0 \sum_{\mathbf{i}} c^\dagger_{\mathbf{i}\uparrow} c^\dagger_{\mathbf{i}\downarrow} + h.c.$$

Deformed interacting Hamiltonian

$$H_\xi \equiv H_0 + \xi (H - H_0)$$

- expand quantities in powers of ξ
- physical answer at $\xi = 1$

pressure $P(\xi) = \frac{1}{\beta L^3} \ln \mathrm{Tr} \exp(-\beta H_\xi) = \sum_{N=0}^{\infty} P_N \xi^N$ $\qquad P(\xi = 1) = P = -\frac{\Omega}{L^3}$

order parameter $\mathcal{O}(\xi) = \langle c_{\mathbf{0}\uparrow} c_{\mathbf{0}\downarrow} \rangle_{H_\xi} = \sum_{N=0}^{\infty} \mathcal{O}_N \xi^N$ $\qquad \mathcal{O}(\xi = 1) = \mathcal{O} = \langle c_{\mathbf{0}\uparrow} c_{\mathbf{0}\downarrow} \rangle$

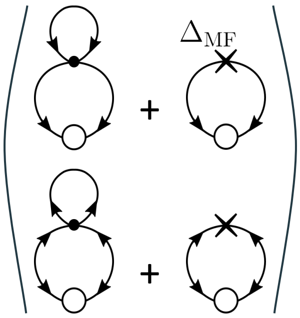
natural choice BCS mean-field theory

$$\mu_{0,\sigma} = \mu_\sigma - U\langle n_{\mathbf{0},-\sigma}\rangle_{H_0}$$

$$\Delta_0 = \Delta_{\rm MF} \equiv -U\langle c_{\mathbf{0}\uparrow} c_{\mathbf{0}\downarrow}\rangle_{H_0}$$

use $\Delta \neq \Delta_{\rm MF}$ as cross-check

$\mathcal{O}_1 =$



$= 0$

Connected Determinant [*R. Rossi, PRL 2017*]

Works with Nambu 2-by-2 propagators

$$\begin{pmatrix} \mathcal{G}_{00}(X-X') & \mathcal{G}_{01}(X-X') \\ \mathcal{G}_{10}(X-X') & \mathcal{G}_{11}(X-X') \end{pmatrix} := -\begin{pmatrix} \langle \mathrm{T}\, c^\dagger_\uparrow(X) c_\uparrow(X')\rangle_{H_0} & \langle \mathrm{T}\, c^\dagger_\uparrow(X) c^\dagger_\downarrow(X')\rangle_{H_0} \\ \langle \mathrm{T}\, c_\downarrow(X) c_\uparrow(X')\rangle_{H_0} & \langle \mathrm{T}\, c_\downarrow(X) c^\dagger_\downarrow(X')\rangle_{H_0} \end{pmatrix}$$

$$\mathcal{O}_N = -\frac{(-U)^N}{N!}\int dX_1 \dots dX_N \ \mathrm{CDet}(A)$$

$$\mathrm{CDet}(A) = \det(A) - \sum(\text{disconnected diagrams})$$

$$X = (\mathbf{i}, \tau)$$

$$A := \begin{pmatrix} 0 & \delta_{\rm sh} & \dots & \mathcal{G}_{00}(X_1-X_n) & \mathcal{G}_{01}(X_1-X_n) & \mathcal{G}_{0\alpha}(X_1) \\ \delta_{\rm sh} & 0 & \dots & \mathcal{G}_{10}(X_1-X_n) & \mathcal{G}_{11}(X_1-X_n) & \mathcal{G}_{1\alpha}(X_1) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathcal{G}_{00}(X_n-X_1) & \mathcal{G}_{01}(X_n-X_1) & \dots & 0 & \delta_{\rm sh} & \mathcal{G}_{0\alpha}(X_n) \\ \mathcal{G}_{10}(X_n-X_1) & \mathcal{G}_{11}(X_n-X_1) & \dots & \delta_{\rm sh} & 0 & \mathcal{G}_{1\alpha}(X_n) \\ \mathcal{G}_{\alpha'0}(-X_1) & \mathcal{G}_{\alpha'1}(-X_1) & \dots & \mathcal{G}_{\alpha'0}(-X_n) & \mathcal{G}_{\alpha'1}(-X_n) & \mathcal{G}_{\alpha'\alpha}(0) \end{pmatrix}$$

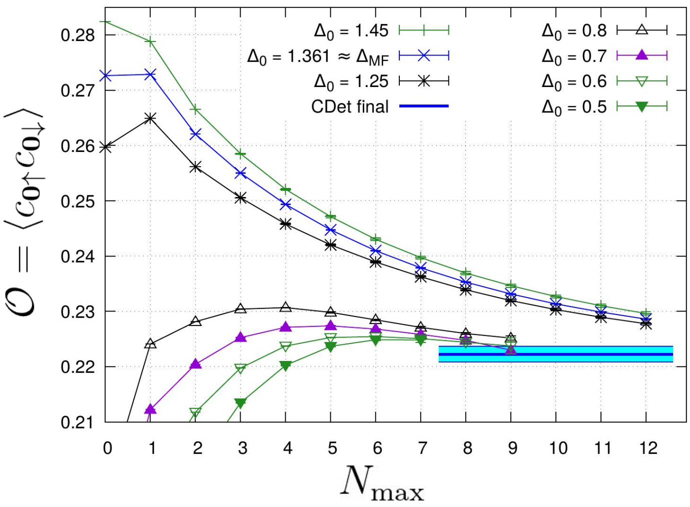
$\alpha = 1, \ \alpha' = 0$

$\delta_{\rm sh} = 0$ if $\Delta_0 = \Delta_{\rm MF}$

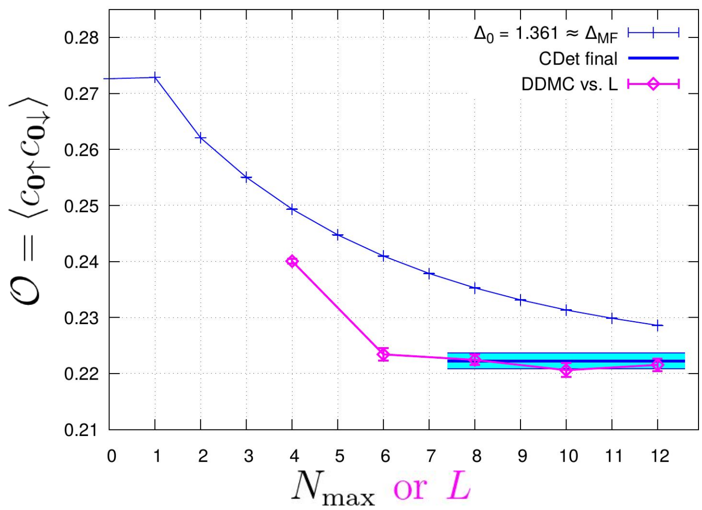
cp. with Determinant Diagrammatic MC [Rubtsov] $\int dX_1 \dots dX_N \det A$

Results in the unpolarized case

$$h = \frac{\mu_\uparrow - \mu_\downarrow}{2} = 0\,, \quad T = 1/8 \approx T_c/2$$



cross-checks and benchmark

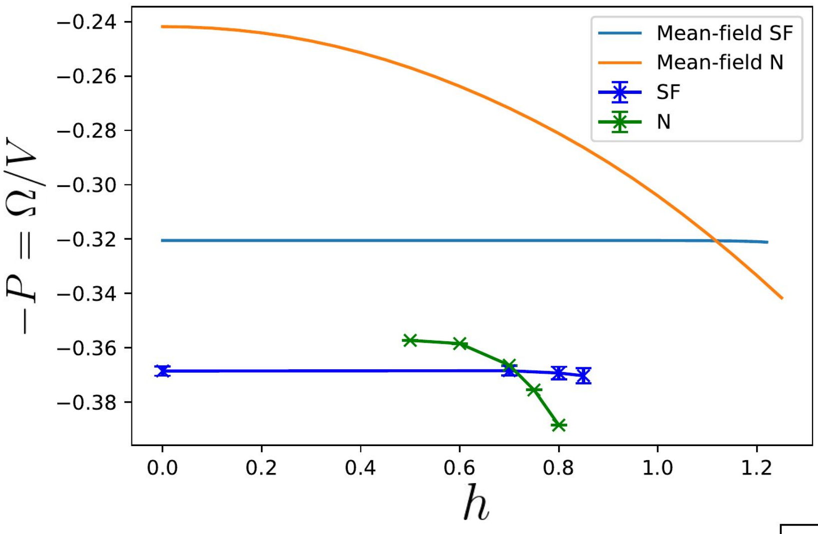


Determinant Diagrammatic MC (E. Burovski's code)

$$|\mathcal{O}_{\rm DDMC}|^2 = \frac{1}{\beta L^3}\int_0^\beta d\tau \sum_i \left\langle (c^\dagger_{\mathbf{i}\uparrow} c^\dagger_{\mathbf{i}\downarrow})(\tau)(c_{\mathbf{0}\uparrow} c^\dagger_{\mathbf{0}\downarrow})(0)\right\rangle$$

Results in the polarized case

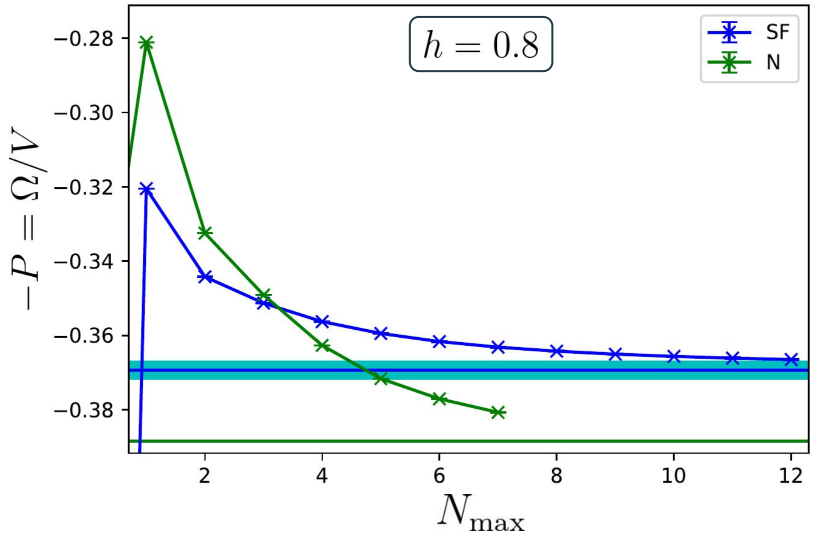
$$T = 1/16 \approx T_c/4$$



SF = Superfluid $\Delta_0 \neq 0$
N = Normal $\Delta_0 = 0$

First order phase transition at the crossing point

for h > 0.7 the normal phase is favored



Large-order behavior of SF expansion

$$h = 0.4\,, \quad T = 1/8 \approx T_c/2\,, \quad \Delta \approx \Delta_{\mathrm{MF}}$$

instanton

$$\mathrm{Disc}\,\mathcal{O}(\xi) \overset{\xi\to 1}{\sim} \exp\left(-\mathrm{const}/\sqrt{1-\xi}\right)$$

$$\mathcal{O}_N \overset{N\to\infty}{\sim} \exp\left(-\mathrm{const} \times N^{\alpha}\right)\,, \quad \alpha < 1$$

Goldstone

$$\mathcal{O}(\xi) \overset{\xi\to 1^-}{\sim} \mathcal{O} + \mathrm{const}\sqrt{1-\xi}$$

$$\mathcal{O}_N \underset{N\to\infty}{\sim} N^{-3/2}$$