Synthetic Flux Attachment

Gerard Valentí-Rojas [1], Niclas Westerberg [2,1], Patrik Öhberg [1]

[1] Institute of Photonics and Quantum Sciences, Heriot-Watt University
[2] School of Physics & Astronomy, University of Glasgow

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CM-CDT The Scottish Doctoral Training Centre in Condensed Matter Physics
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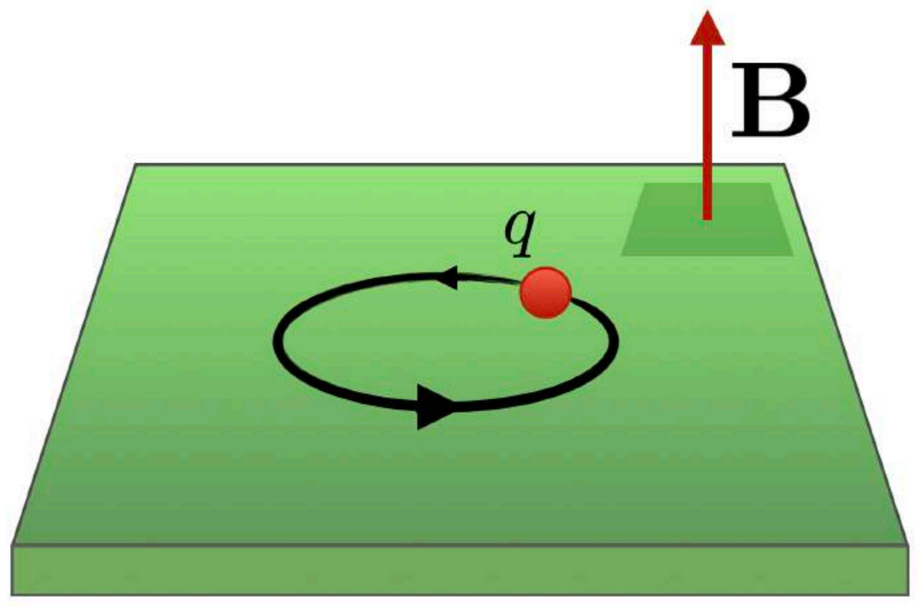
(1)

Is this a gauge theory ?

NO! Gauge field is Background (non-dynamical)

Charged particle in a magnetic field

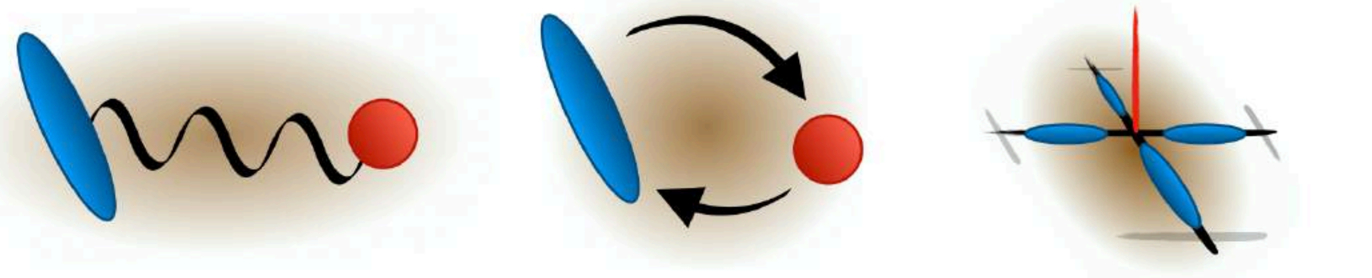


$$H = \frac{(\mathbf{p} - q\mathbf{A})^2}{2m}$$

(2)

Recipe for a gauge theory

A backaction mechanism is needed between gauge and matter sectors

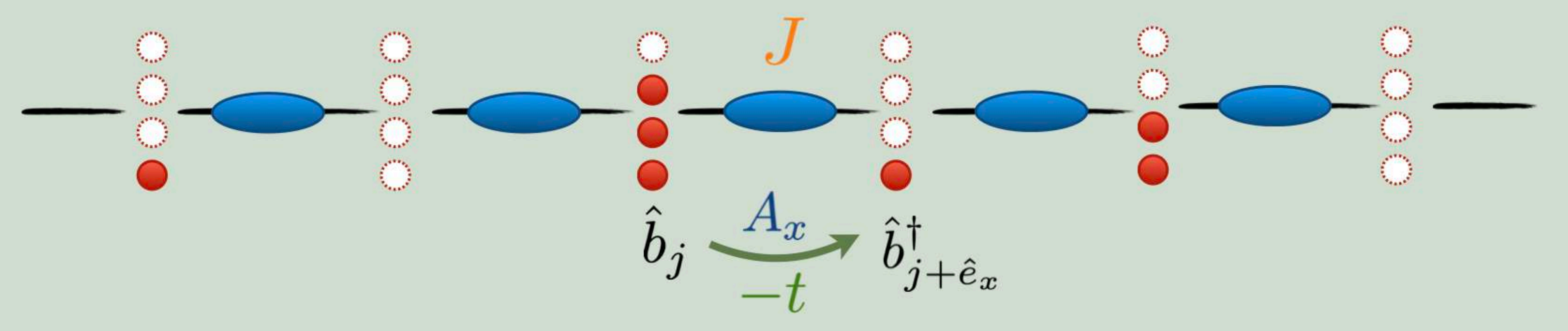


(i) Gauge-Matter Coupling

(ii) Dynamical Gauge fields

(iii) Local Gauge Invariance

Some random example: Lattice "Scalar Electrodynamics" in 1+1D



Impose Local Constraint

$E_{j+1} - E_j = (N\,●)_j$ — To continuum → $\vec{\nabla} \cdot \mathbf{E} = \rho$

$$H = -t \sum_j (\hat{b}^\dagger_{j+\hat{e}_x}\, e^{iA_x}\, \hat{b}_j + \text{H.c.}) + J \sum_j E_j^2$$

where $E_j = -\partial_t\, A_x(j)$

How to go from a theory of background gauge fields **(1)** to a gauge theory **(2)** ? Supplement **(1)** with some constraint!

F. Görg et al., Nature Phys. 15, 1161 (2019)
V. Lienhard et al., Phys. Rev. X 10, 021031 (2020)
C. Schweizer et al., Nature Phys. 15, 1168 (2019)

$$i\hbar\frac{\partial}{\partial t}\Psi(t,\mathbf{r}) = -\frac{\hbar^2}{2m}\left(\nabla - i\frac{e}{\hbar}\mathbf{A}\right)^2\Psi(t,\mathbf{r})$$

+

$$A_i\left(t,\mathbf{r}\right) = f\left[n\left(t,\mathbf{r}\right)\right]\hat{e}_i \quad \text{where} \quad n\left(t,\mathbf{r}\right) = |\Psi|^2$$

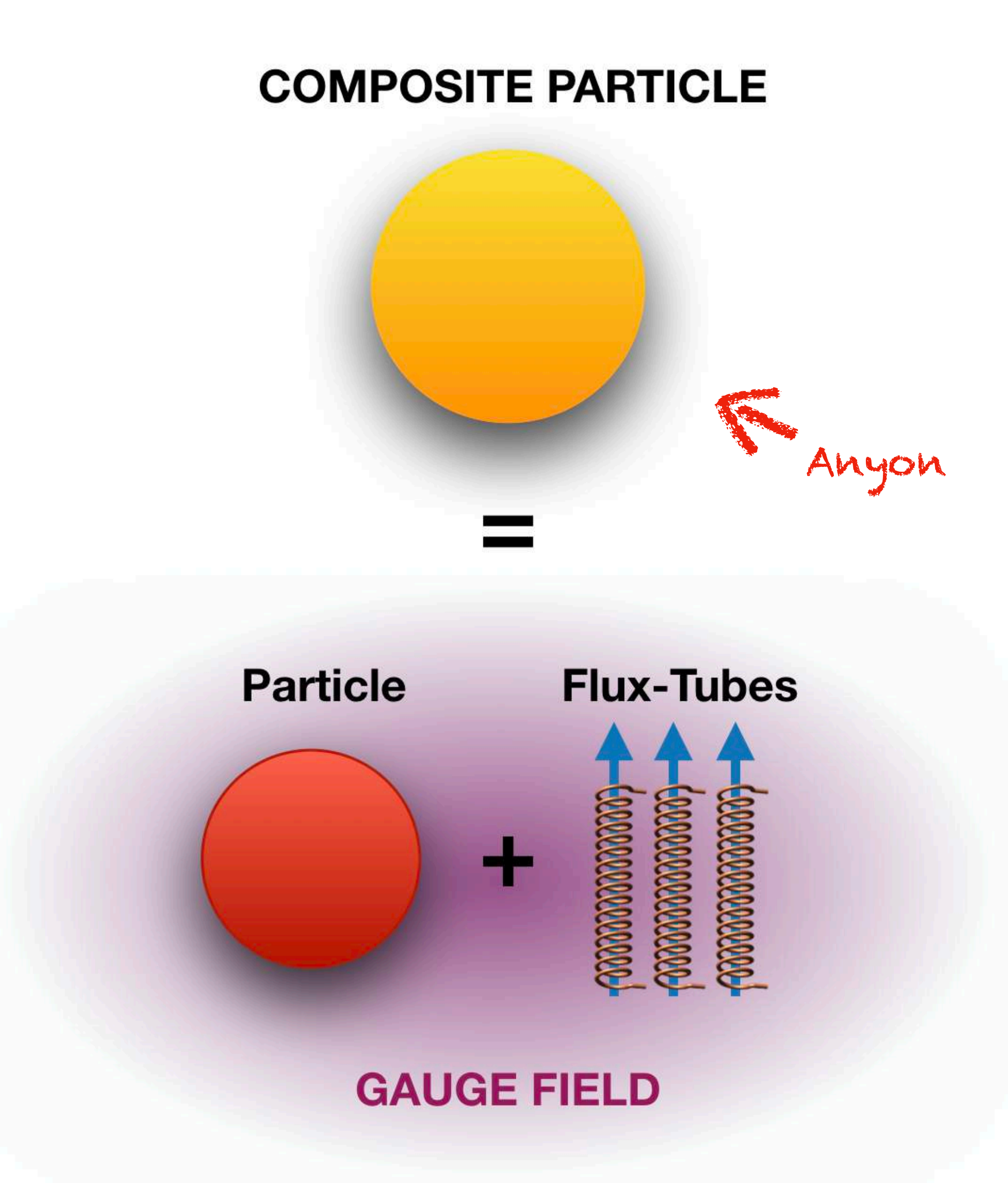
Gauge field is some function of matter density

Example in 2D: Flux Attachment / Chern-Simons Theory / Fractionalisation

Flux attachment is a mechanism by which charged particles capture magnetic flux quanta and form composite entities. As a consequence of flux dressing, these composites may acquire fractional quantum numbers and statistics.

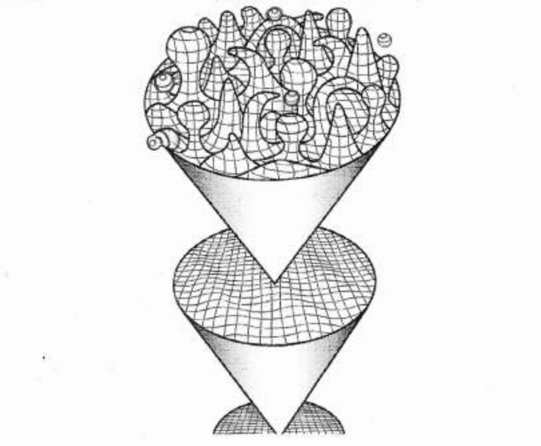
- Start from a Chern-Simons Term $\mathcal{L}_{CS} = -\frac{\kappa}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda}$ — Note this is similar to a Maxwell term but with one derivative less
- Equation of motion with a matter source $\frac{\kappa}{2\pi}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} = j^\mu$ — (Generic) matter content is found in the current
- Write in components and solve $B \equiv \nabla\times\mathbf{A} = \frac{2\pi}{\kappa}\rho$ → Coulomb Gauge → $\mathbf{A}(\mathbf{r}) = \frac{1}{\kappa}\left(\hat{z}\times\int d^2\mathbf{r}'\,\frac{\mathbf{r}-\mathbf{r}'}{|r-r'|^2}\rho(\mathbf{r}')\right)$

Flux Attachment | **Density-dependent Gauge Field**



GOAL: From a **microscopic** interacting quantum-many body system. Derive "the emergence" of a Chern-Simons term so that it performs flux attachment at an effective level.

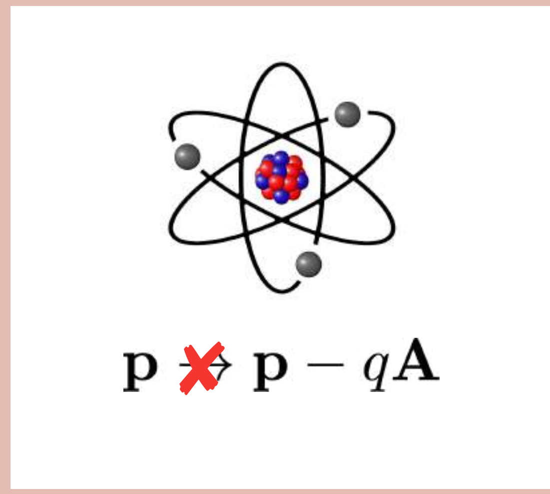
Challenges



Flux Attachment is Emergent

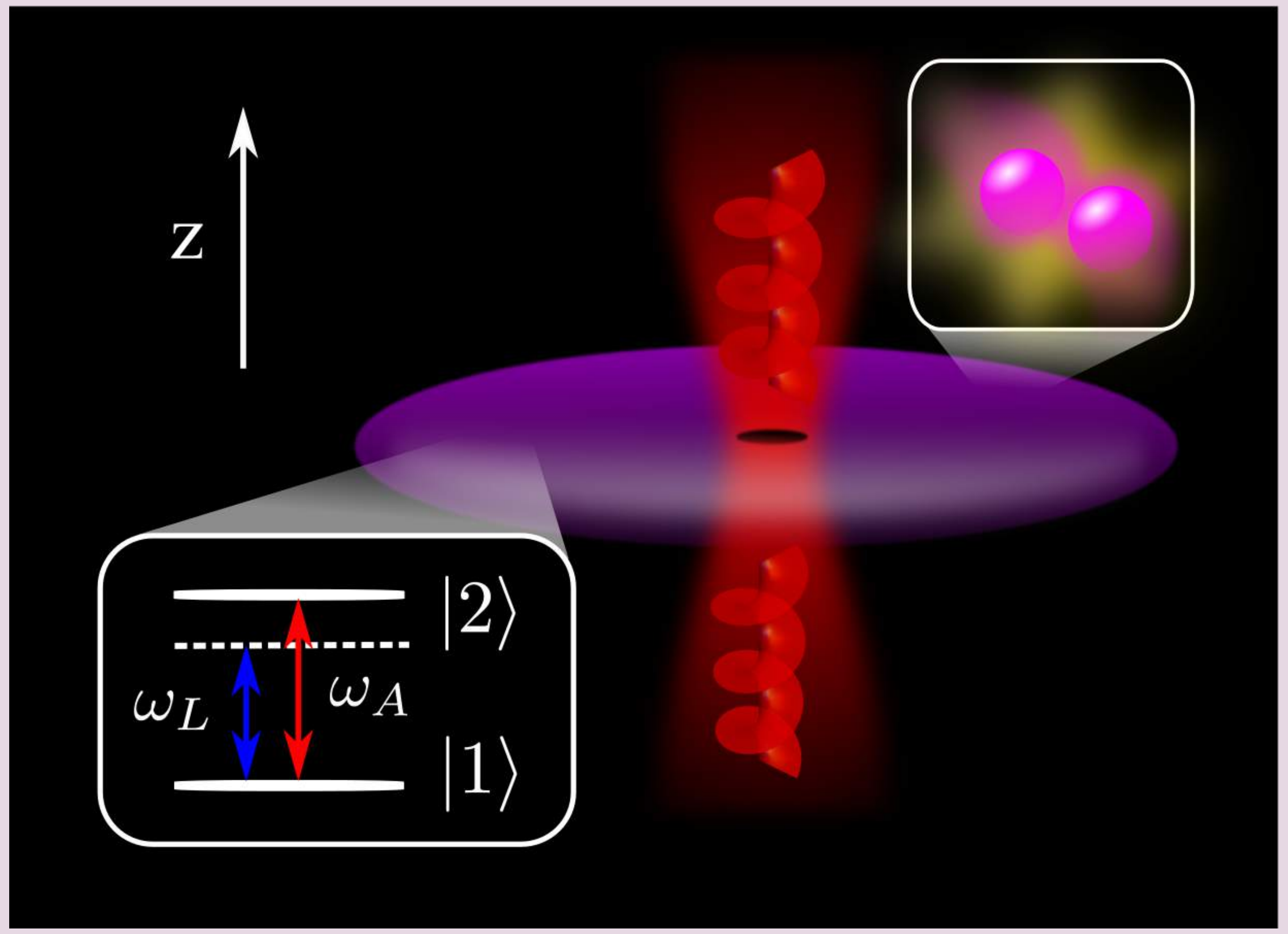
$$H_{CS} = 0$$

It is a Topological Field Theory



Ultracold Atoms: Dilute & Charge Neutral

Microscopic Scheme



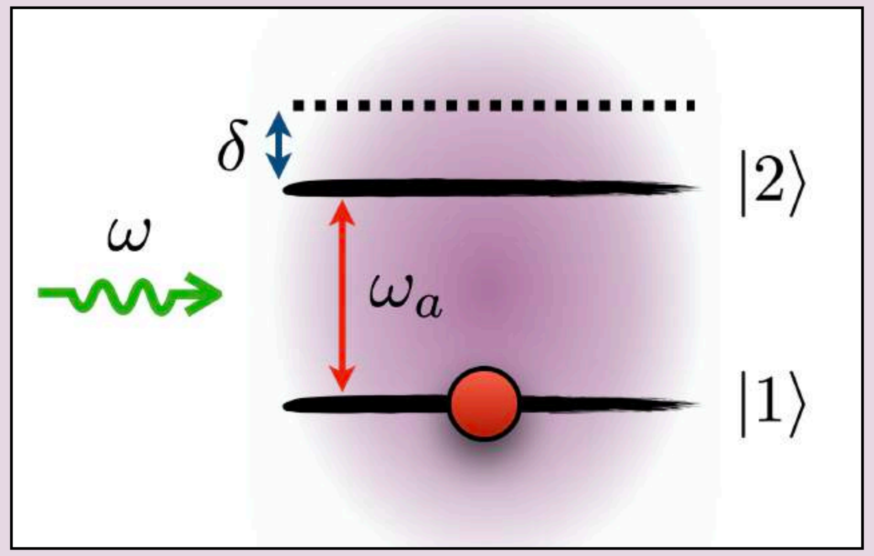
BEC of atoms with 2 internal levels coupled by a laser beam

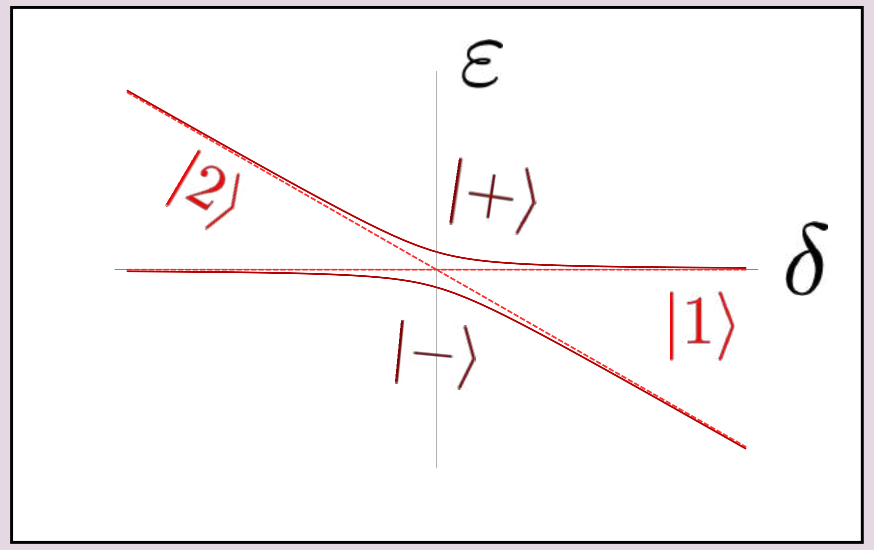
$$H=\sum_i\left(\frac{\mathbf{p}_i^2}{2m}+V_{\text{ext}}(\mathbf{r}_i)+U(\mathbf{r}_i)\right)+\sum_{\sigma,\sigma'=1}^{2}\sum_{i<j}g_{\sigma\sigma'}\,\delta\left(\mathbf{r}_i-\mathbf{r}_j\right)$$

Interparticle contact pairwise Interaction $\mathcal{V}_{ij}$

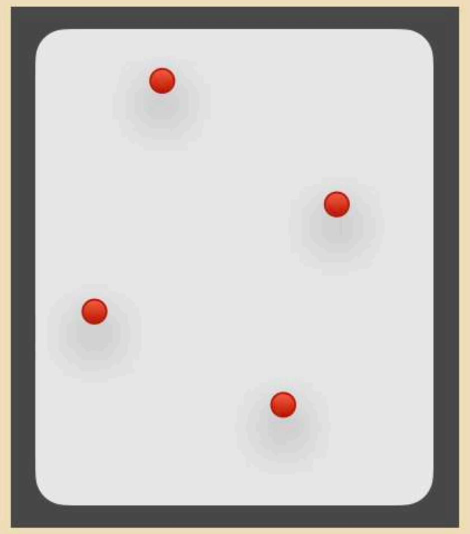
External Potential e.g. trapping potential

Light-Matter Interaction $U(\mathbf{r}_i)=\hbar\Omega(\mathbf{r}_i)\left(\mathbf{n}(\mathbf{r}_i)\cdot\vec{\sigma}\right)$





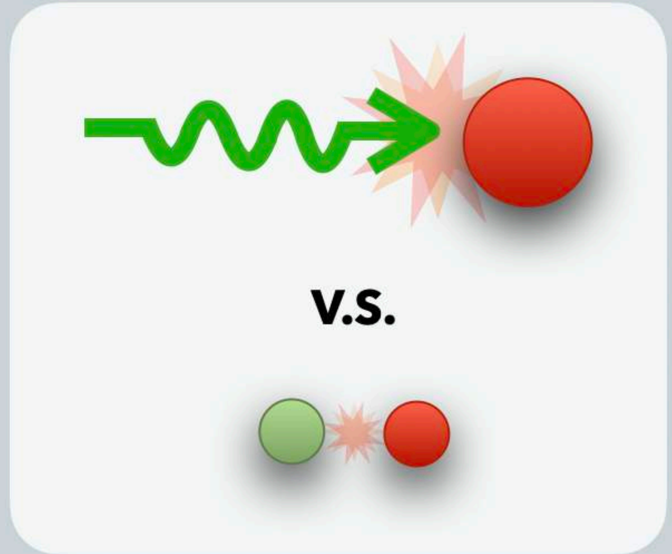
Approximations: "Deriving" emergence



$\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_N)\approx\tilde{\Psi}(\mathbf{r}_1)\ldots\tilde{\Psi}(\mathbf{r}_N)$

Mean-Field

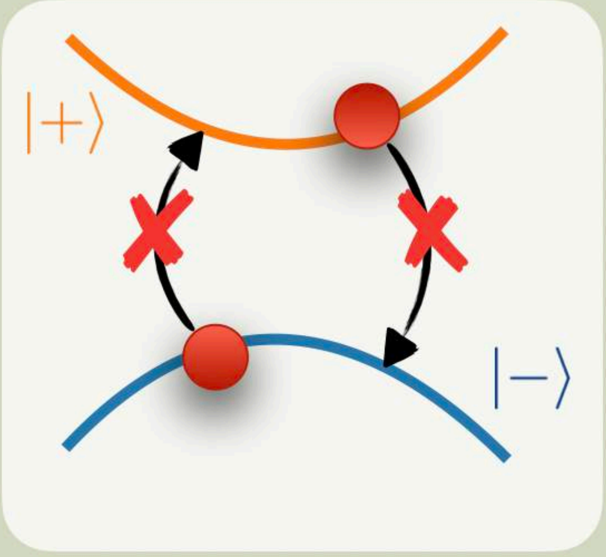
$H_{\text{MF}}=\frac{\mathbf{p}^2}{2m}+V+U+\mathcal{V}_{\text{MF}}$



$\Omega\gg g$

Perturbation

$|\pm\rangle\approx|\pm^{(0)}\rangle+|\pm^{(1)}\rangle$



$|\tilde{\Psi}(t,\mathbf{r})\rangle\approx\Phi_\pm(t,\mathbf{r})\,|\pm(\mathbf{r})\rangle$

Adiabatic

$i\hbar\,\partial_t\,\Phi_\pm=H_{\text{eff}}^{\pm}\,\Phi_\pm$

Mean-field Hamiltonian is projected onto the eigenstate in which the system is prepared

Effective model: A topological gauge theory in 2+1D

$$\mathcal{L}_{\text{eff}}\approx-\frac{\kappa}{4\pi\hbar}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda+i\hbar\,\Phi^*D_t\Phi-\frac{\hbar^2}{2m}|\mathbf{D}\Phi|^2-\frac{g}{2}|\Phi|^4-\tilde{W}|\Phi|^2$$

$$D_\mu=\partial_\mu-\frac{i}{\hbar}\left(A_\mu+a_\mu\right)\qquad\text{where}\qquad\mu=t,x,y$$

Berry Connection

$$i\hbar\,\langle+|\,\vec{\nabla}\,|+\rangle\approx\mathbf{A}^{(0)}+\mathbf{A}^{(1)}=\vec{A}+\vec{a}$$

Perturbative Expansion

Background gauge field
Single-Particle contribution

(Dynamical) Chern-Simons gauge field
Interaction (two-body) contribution

Phenomenology

Effective Model Corresponds to

Macroscopic or Composite Boson description of a FQH fluid

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Effective-Field-Theory Model for the Fractional Quantum Hall Effect

S. C. Zhang
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

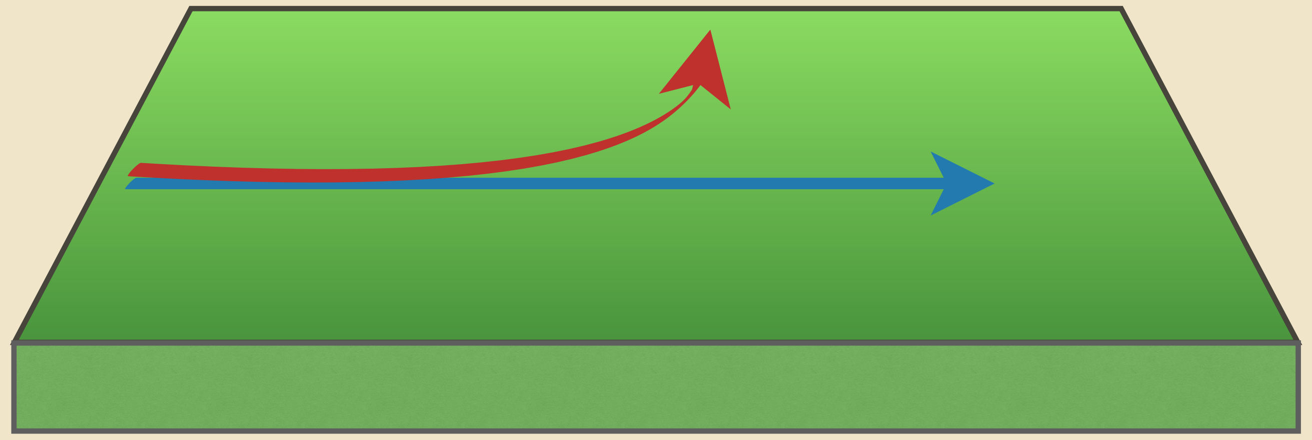
T. H. Hansson and S. Kivelson
Physics Department, State University of New York at Stony Brook, Stony Brook, New York 11794
(Received 26 July 1988)

Super Nice Review !

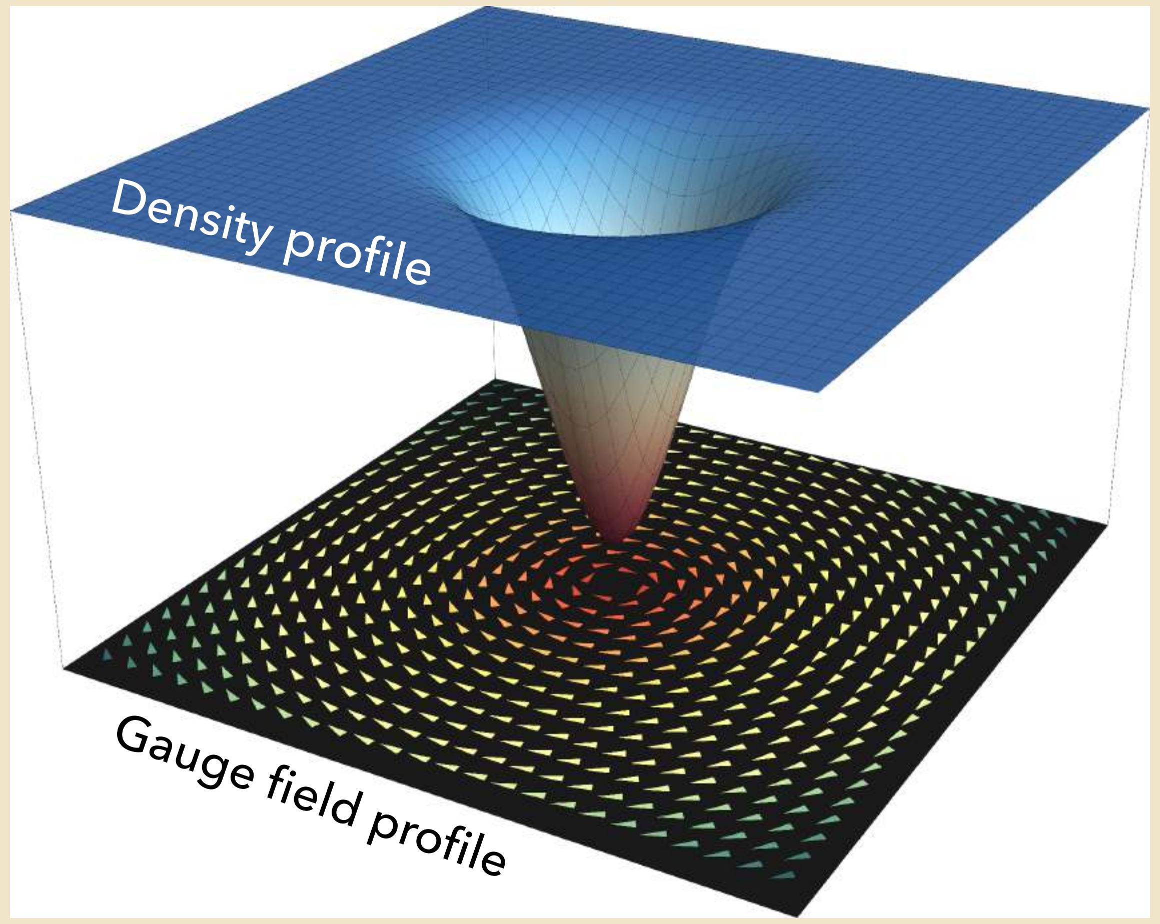
International Journal of Modern Physics B, Vol. 6, No. 1 (1992) 25–58

THE CHERN–SIMONS–LANDAU–GINZBURG THEORY OF THE FRACTIONAL QUANTUM HALL EFFECT*

SHOU CHENG ZHANG
IBM Research Division, Almaden Research Center, 650 Harry Road, San Jose, CA 95120-6099, USA



Fractionally quantised (atomic) Hall conductance and transverse flow



Flux attached vortices

- Fractional (synthetic) charge
- Anyonic statistics

They act as Laughlin quasiparticles

Summary & Conclusions

- We derive emergent topological gauge theory in **2+1D** in **continuum**. Chern-Simons gauge field is understood as a **density-dependent** Berry connection (synthetic gauge field)
- We introduce a proof-of-concept scheme for a potential quantum simulation using a BEC. Only **one species needed** as compared to two species used in conventional LGTs
- We obtain an effective (strongly correlated) FQH fluid with fractionalised excitations (vortices) out of a dilute weakly interacting system. We “induce” flux attachment
- Systems with density-dependent gauge fields can be understood as gauge theories with (certain) topological structure
- Discretisation of the model for a lattice realisation is straightforward. Extensions as coupling to fermions or higher-spin structures are a subject for further work