Supersolid Stripe Crystal from Finite-Range Interactions on a Lattice

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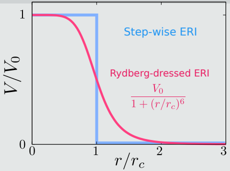
The search for phases of matter where exotic states may be stabilized by the simultaneous breaking of different symmetries is a subject of central interest in condensed matter physics. A prominent example is supersolidity (i.e. coexistence of superfluidity and cristalline order)[1]. In this context, a large class of Extended-range Repulsive (pair-wise) Interactions (ERI) has recently elicited considerable scientific attention . ERI are of immediate interest for experiments employing Rydberg-dressed atoms[2]. At high enough densities ERI are characterized by clusterization, a feature that has been shown to be linked to supersolidity in two- dimensional continuous space [3] and supersolidity and (super)glassines (the latter in the absence of extenal frustration) on a triangular lattice [4]. Here we are interested in studying the ground state phases of monodisperse bosonic particles on a square lattice interacting via ERI of soft-shoulder type.

Extended Range Interactions (ERI)



- Rydberg dressed-atoms[7]
- Experimentally realized[2]
- Superglass (glassy + superfluid) phase on a triangular lattice[4]

Supersolid formation with ERI

- Free space tunneling of particle between superfluid droplets[5]
- **NEW** Cluster self-assembly + superfluid exchanges between clusters on a lattice[6] (this poster!)

[1] Boninsegni, M. *et al.* *Rev. Mod. Phys.* **84,** 759–776 (May 11, 2012); [2] Henkel, N. *et al.* *Phys. Rev. Lett.* **104,** 195302 (May 11, 2010), Jau, Y.-Y. *et al.* *Nature Physics* **12,** 71–74 (Jan. 2016), Zeiher, J. *et al.* *Nature Physics* **12,** 1095–1099 (Dec. 2016); [3] Cinti, F. *et al.* *Nature Communications* **5,** 3235 (Feb. 4, 2014); [4] Angelone, A. *et al.* *Phys. Rev. Lett.* **116,** 135303 (Apr. 1, 2016); [5] Cinti, F. *et al.* *Phys. Rev. Lett.* **105,** 135301 (Sept. 21, 2010); [6] Masella, G. *et al.* *Phys. Rev. Lett.* **123,** 045301 (July 26, 2019); [7] Pupillo, G. *et al.* *Phys. Rev. Lett.* **104,** 223002 (June 1, 2010), Johnson, J. E. *et al.* *Phys. Rev. A* **82,** 033412 (Sept. 14, 2010);

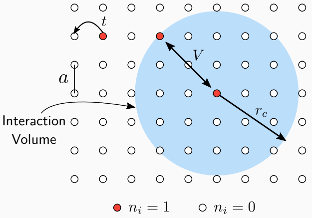
Model Hamiltonian

We study hard-core bosons on a square lattice:

$$\mathscr{H} = \underbrace{-t \sum_{\langle i,j \rangle} \left(b_i^\dagger b_j + \text{h.c.} \right)}_{\text{Hopping}} + \underbrace{V \sum_{i<j;r_{ij} \le r_c} n_i n_j}_{\text{Potential}}$$

Cluster formation for $r_c > 1$ and high enough density ρ.

We study the case of $\rho = 5/36$ and $r_c = 2\sqrt{2}$.



Methods and Observables

We determine the ground state phases of the proposed model by means of Path integral Monte Carlo simulations based on the Worm Algorithm[1].

The Worm algorithm is a numerically exact technique when applied to unfrustrated bosonic models

We compute the following observables:

- Superfluidity $\longrightarrow$ Superfluid Fraction

$$\rho_s/\rho = \frac{\langle W_x^2 + W_y^2 \rangle}{4t\rho\beta} \qquad (W_{x,y} \equiv \text{winding number})$$

- Crystalline structure $\longrightarrow$ Structure Factor

$$S(\mathbf{k}) = \frac{\left\langle \sum_{i,j} e^{i\mathbf{k}\cdot\mathbf{r}_{ij}} n_i n_j \right\rangle}{N^2} \qquad (\mathbf{k} \equiv \text{lattice momentum})$$

- Long-range off-diagonal order $\longrightarrow$ Green Function

$$G(\mathbf{r}) = \frac{\left\langle \sum_i b_i^\dagger b_{i+\mathbf{r}} \right\rangle}{N} \qquad (N \equiv \text{number of sites})$$

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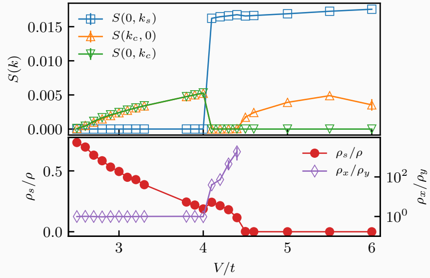
[1] Prokof'ev, N. V. *et al.* *J. Exp. Theor. Phys.* **87,** 310–321 (Aug. 1, 1998)

Stripe Supersolids and Crystals

We investigate the ground state phase diagram of our model as a function of the interaction strength V/t. We find [2]:



- Low-V: Superfluid (SF)
- High-V: Stripe Crystal (SC)
- Intermediate-V:
  - Isotropic Supersolid (IS)
  - Stripe Supersolid (SS)

Supersolid-Supersolid transition

Figure on the left: Panel (a)Structure factor components for the isotropic and anisotropic orders as a function of the interaction strength V/t Panel (b) Superfluid fraction and ratio between superfluid responses as a function of V/t.

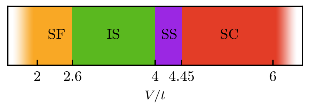


Figure: Ground state phase diagram

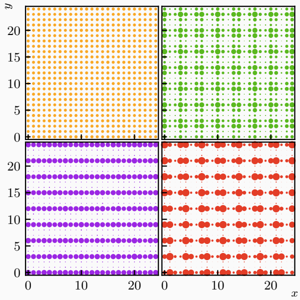


Figure: Averaged site-density maps. The size of the dots is proportional to the occupation of the corresponding lattice site. Colors match the phases above

Isotropic Supersolid

- Isotropic long-range oerder with $S(\mathbf{k}$ peaks at $\mathbf{k} = (0, \pm k_c), (\pm k_c, 0)$, $k_c = 2\pi\frac{7}{24}$.
- Isotropic superfluid exchanges

Anisotropic Supersolid

- Crystalline order only on y-axis with $S(\mathbf{k})$ peaks at $\mathbf{k} = (0, \pm k_c), (\pm k_c, 0)$, $k_c = 2\pi\frac{7}{24}$.
- Isotropic superfluid exchanges

[2] Masella, G. *et al.* *Phys. Rev. Lett.* **123,** 045301 (July 26, 2019)

Out-of-equilibrium scenarios

(Super)Glass

- Frozen degrees of freedom + lack of structural order
- Finite Edwards-Anderson parameter[1]
  $\tilde{q}_{\mathrm{EA}} = \left(\sum_i \langle n_i - \rho \rangle^2\right) / (N\rho(1-\rho))$
- Superglass: glassy behaviour + superfluidity

We make use of simulated quenches from $T \to \infty$ configurations to drive the system out-of-equilibrium (OOE).

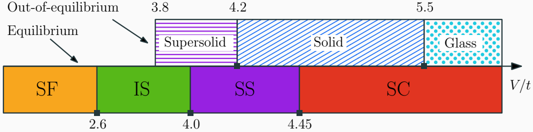


Figure: Equilibrium phases and out-of-equilibrium (OOE) states as a function of V/t.

We find[2]

- Always isotropic OOE (super) solid states
- Equilibrium crystal → OOE Glass
- No Superglass

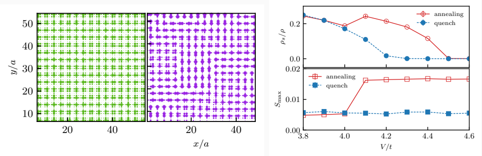


Figure: (left) Density snapshots of equilibrium and OOE isotropic (super)solids, (right) comparison of annealing (equilibrium) and quench (OOE) values for the superfluid density (top) and maximum structure factor (bottom) as functions of V/t.

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[1] Carleo, G. *et al.* *Phys. Rev. Lett.* **103,** 215302 (Nov. 18, 2009)

[2] Angelone, A. *et al.* *Phys. Rev. A* **101,** 063603 (June 1, 2020)